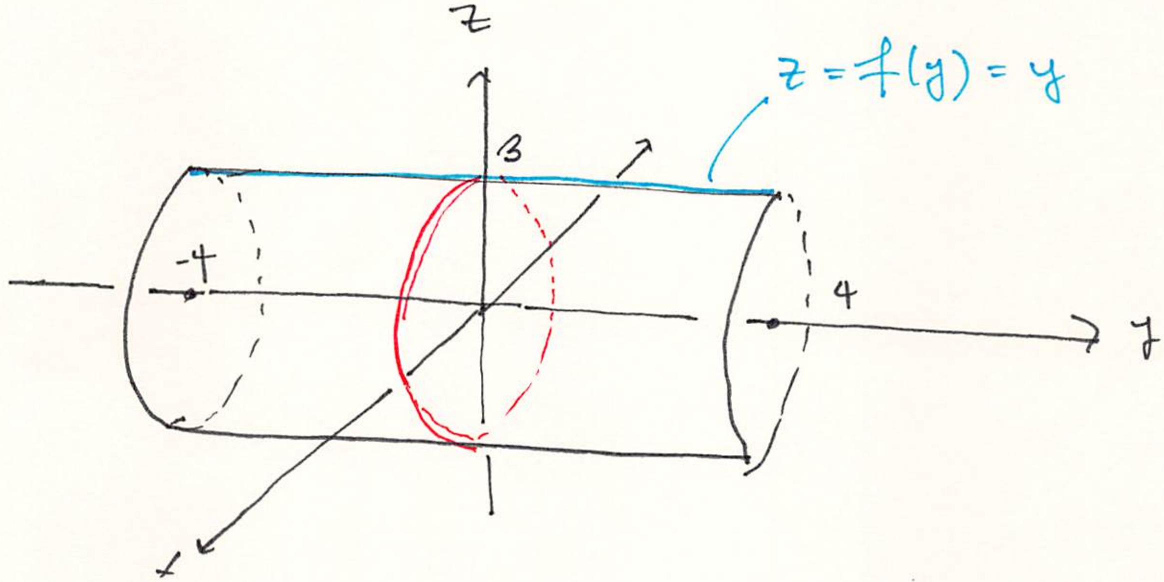
16.6

#24. $x^2+z^2=9$, above xy-plane.

between $y=-4$ & $y=4$



Surface of revolution of $z=f(y)=y$

over $-4 \le y \le 4$

$$\vec{r}(y,\theta) = f(y)\cos\theta\,\vec{i} + y\,\vec{j} + f(y)\sin\theta\,\vec{k}$$

$$= y\cos\theta\,\vec{i} + y\,\vec{j} + y\sin\theta\,\vec{k}$$

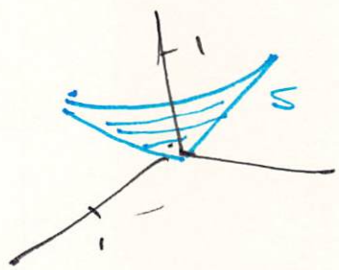
16.7

#6. $\iiint_S xyz\,dS$, S is the cone

$\vec{r}(u,v) = \langle u\cos v, u\sin v, u \rangle$

$0 \le u \le 1$

$0 \le v \le \pi/2$



$\vec{r}_u = \langle \cos v, \sin v, 1 \rangle$

$\vec{r}_v = \langle -u\sin v, u\cos v, 0 \rangle$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$$= \langle -u\cos v, u\sin v, u \rangle$$

$$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{u^2\cos^2 v + u^2\sin^2 v + u^2}$$

$$= \sqrt{2u^2}$$

$$= \sqrt{2}\,|u| \qquad (0 \le u \le 1)$$

$$= \sqrt{2}\,u$$

$$f(x,y,z) = xyz$$

$$\Rightarrow f(\vec{r}(u,v)) = (u\cos v)(u\sin v)(u)$$

$$= u^3\cos v\sin v$$

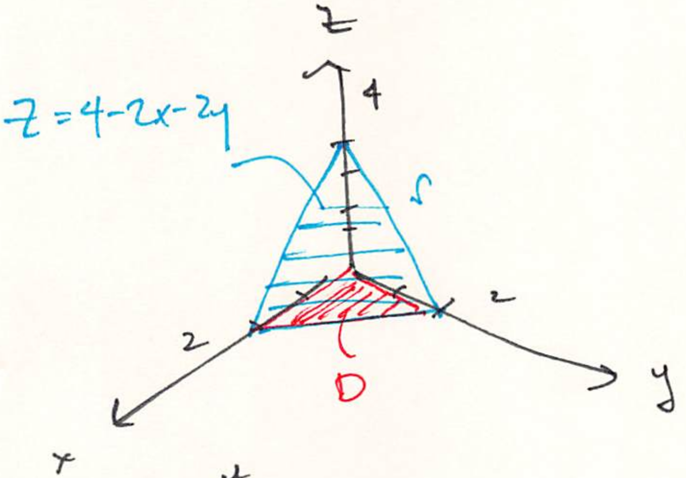
$$\therefore \iint_S xyz\,dS = \int_0^{\pi/2}\int_0^1 u^3\cos v\sin v \cdot \sqrt{2}\,u\,du\,dv$$

$$= \sqrt{2}\int_0^1 u^4\,du \cdot \int_0^{\pi/2}\cos v\sin v\,dv$$

$$= \sqrt{2}\cdot\frac{1}{5}\cdot\int_0^1 u\,du$$

$$= \frac{\sqrt{2}}{10}$$

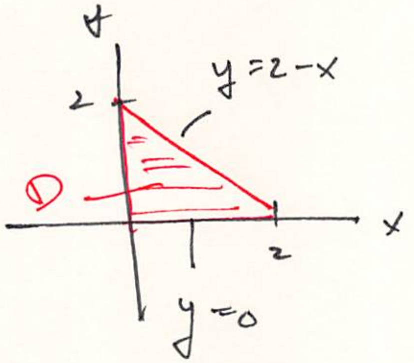
#10. $\iint_S xz\,dS$, part of $2x+2y+z=4$ in first octant.



$$\vec{r}(x,y) = \langle x, y, 4-2x-2y \rangle$$

D: $0 \le y \le 2-x$

$0 \le x \le 2$



$$\vec{r}_x = \langle 1, 0, -2 \rangle$$

$$\vec{r}_y = \langle 0, 1, -2 \rangle$$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= \langle 2, 2, 1 \rangle$$

$$\|\vec{r}_x \times \vec{r}_y\| = \sqrt{4+4+1} = 3$$

$$f(x,y) = xz$$

$$\Longrightarrow f(\vec{r}(x,y)) = x\,(4-2x-2y)$$

$$= 4x - 2x^2 - 2xy$$

$$\therefore \iint_S xy\,dS = \int_0^2 \int_0^{2-x} (4x-2x^2-2xy)\, 3\, dy\, dx$$

$$= 3\int_0^2 \Big[(4x-2x^2)y - xy^2\Big]_0^{2-x} dx$$

$$= 3\int_0^2 +2x\,(2-x)(2-x) - x(2-x)^2\, dx$$

$$= 3\int_0^2 (2-x)^2\, dx \qquad \begin{aligned} u &= 2-x \\ du &= -dx \\ u(0) &= 2 \\ u(2) &= 0 \end{aligned}$$

$$= 3\int_0^2 u^2\, du$$

$$= u^3\Big|_0^2 = 8$$