Math 244 Exam 2, Fall 2023

Name:

Question	Points	Score
1	9	
2	18	
3	10	
4	9	
5	0	
Total:	46	

- You have 50 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 3x5 notecard with notes, no other resources are authorized.
- You may use a scientific calculator, no other electronic devices are authorized.
- The back side of each page can be used as scratch paper.

Homework	
Exam 1	
Exam 2	
Total	

1. (9 points) Let R be a region in the xy-plane and suppose $f(x, y)$ is a function. Let $T(u, v) = (x(u, v), y(u, v))$ be a transformation with the following properties:

   (i) The Jacobian of T is $-3/17$.

   (ii) T maps the rectangle $1 \leq u \leq 2$ and $0 \leq v \leq 3$ to the region R in the xy-plane.

   (iii) $f(T(u, v)) = ue^{uv}$.

   Evaluate the integral $\iint_R f(x, y)dA$.

2. (18 points) Let E be the solid the lies inside the cylinder $x^2 + y^2 = 1$, below the paraboloid $z = x^2 + y^2$, and above the xy-plane.

   (a) Set up the volume triple integral in rectangular coordinates.

   (b) Set up the volume triple integral in cylindrical coordinates.

   (c) Find the volume of E.

3. (10 points) Let E be the spherical wedge in the first octant cut from unit ball by the planes $y = 0$ and $y = x$.

   (a) Sketch the region.

   (b) Convert the following integral to spherical coordinates

$$\iiint_E \exp(-x^2 - y^2 - z^2)\, dV.$$

   Do not evaluate the integral.

4. (9 points) Consider the integral

$$\int_0^2 \int_0^1 \int_0^{y^2} f(x, y, z)\, dzdydx.$$

(a) Sketch the region of integration.

(b) Change the order of integration to $dydxdz$.

5. (5 points (bonus)) Give an example of a region R, a function $f(x, y)$, and a transformation $T(u, v)$ satisfying the hypothesis of problem 1.