Math 244 Exam 3, Fall 2023

Name:

Question	Points	Score
1	8	
2	13	
3	19	
4	0	
Total:	40	

- You have 50 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 3x5 notecard with notes, no other resources are authorized.
- You may use a scientific calculator, no other electronic devices are authorized.
- The back side of each page can be used as scratch paper.

Homework	
Exam 1	
Exam 2	
Exam 3	
Total	

1. (8 points) Evaluate the line integral

$$\int_C \frac{y}{x}\, ds,$$

where C is the path parametrized by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, $1 \leq t \leq 2$.

2. Suppose

$$\nabla f = \langle e^x \cos y + yz, xz - e^x \sin y, xy + z \rangle.$$

(a) (7 points) Find the potential f that satisfies $f(0,0,0) = 1$.

(b) (3 points) Find $\nabla \times \nabla f$.

(c) (3 points) Find $\oint_C \nabla f \cdot d\mathbf{r}$ where C is the closed curve on the unit sphere parametrized by

$$r(t) = \langle \cos t, \sin t \cos t, \sin^2 t \rangle, \quad 0 \leq t \leq 2\pi.$$

3. Let

$$\mathbf{F} = \langle x + y, y - x \rangle,$$

and let D be the region inside the unit circle that lies above the x-axis with its boundary ∂D oriented counterclockwise.

(a) (13 points) Evaluate the line integral $\int_{\partial D} \mathbf{F} \cdot d\mathbf{r}$ directly by parametrizing ∂D.

(b) (6 points) Verify your answer to part (a) using Green's theorem.

(more space)

4. (5 points (bonus)) Sketch the curve in (2c).