Math 244 Final, Fall 2023

Name:

Question	Points	Score
1	14	
2	10	
3	13	
4	23	
5	9	
6	9	
7	11	
8	5	
Total:	94	

- You have 120 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 8x5 notecard with notes, no other resources are authorized.
- You may use a scientific calculator, no other electronic devices are authorized.
- The back side of each page can be used as scratch paper.

1. (14 points) Find the outward flux of $\mathbf{F} = \langle x^3, y^3, z^3 \rangle$ over the unit sphere $x^2+y^2+z^2 = 1$.

2. (10 points) Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

3. (13 points) Find the moment of inertia of a homogeneous right circular cylinder of radius a and height h about its axis.

4. Let $\mathbf{F} = \langle x^2 z, 3x, y^3 \rangle$ and let S be the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 0$ and $z = 1$, oriented upward.

(a) (13 points) Evaluate $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ directly using the parametrization

$$\mathbf{r}(z, \theta) = \langle z \cos\theta, z \sin\theta, z \rangle, \quad 0 \le z \le 1, \quad 0 \le \theta \le 2\pi.$$

(b) (10 points) Verify your answer to part (a) using Stokes' theorem.

(more space)

5. A force field is given by $\mathbf{F} = \langle e^y, xe^y + \sin z, y \cos z \rangle$.

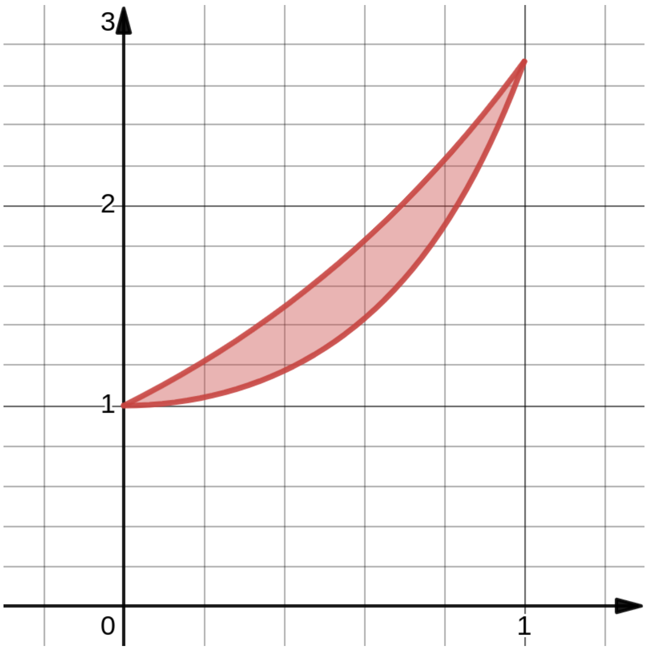
   (a) (6 points) Find a potential function for $\mathbf{F}$.

   (b) (3 points) Find the work done by $\mathbf{F}$ on a particle that moves from $(1, 0, 7)$ to $(-3, 0, 9)$.

6. (9 points) Evaluate the line integral

$$\int_C x \ln y \, dx,$$

where C is the boundary of the region $0 \le x \le 1$ and $e^{x^2} \le y \le e^x$.



7. (11 points) Find the surface area of the portion of the helicoid parametrized by

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 2\pi.$$

8. (5 points) Hyperbolic sine and cosine are defined as

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \text{ and } \cosh(x) = \frac{e^x + e^{-x}}{2},$$

respectively. *Elliptic coordinates* are given by the relations

$$\begin{aligned} x &= \cosh(u)\cos(v) \\ y &= \sinh(u)\sin(v) \end{aligned}$$

Find the Jacobian determinant of this transformation, you do not have to simplify your answer.

For **5 bonus points** simplify your answer to

$$\sinh^2(u) + \sin^2(v) \text{ or } \cosh^2(u) - \cos^2(v).$$