

Math 244 Exam 3, Spring 2022

Name:

Question	Points	Score
1	10	
2	14	
3	17	
4	10	
5	0	
Total:	51	

- You have 50 minutes to complete this exam.
- All work must be your own.
- You may use a 3x5 notecard with notes that you may turn in for extra credit.
- You must show all your work. You will get almost no credit for solutions that are not fully justified.
- Answer the questions in the space provided. Use the back of the page for scratch work or if you require additional space for your answers. Clearly indicate what is a solution, and what is scratch work.
- No electronic devices are authorized with the exception of a scientific calculator.
- Good luck!

1. (10 points) A thin wire C with a density $\rho(x, y) = y$ is parametrized by $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$ where $0 \leq t \leq \sqrt{2}$. Find the mass of the wire.

2. (14 points) Find the work done by the force field $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + xy\mathbf{k}$ on a particle that moves along the path parametrized by $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t/2\mathbf{k}$ where $0 \leq t \leq 3\pi/2$.

3. Throughout this problem let $\mathbf{F} = 3x^2\mathbf{i} + \frac{z^2}{y}\mathbf{j} + 2z \ln y\mathbf{k}$.

(a) (5 points) Show that \mathbf{F} is conservative.

(b) (9 points) Find a potential function for \mathbf{F} .

(c) (3 points) Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the ellipse where the cylinder $x^2 + z^2 = 4$ and the plane $y = 5 - x$ meet, oriented counterclockwise when viewed from above the xz -plane.

(continue problem 3 here)

4. (10 points) Let C be the unit circle oriented counterclockwise. Use Green's theorem to evaluate the line integral

$$\oint_C (2x^3 - y^3)dx + (x^3 + y^3)dy.$$

5. (5 points (bonus)) Let

$$\mathbf{F} = \frac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2}.$$

Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the positively oriented ellipse $9x^2 + 4y^2 = 36$.