

Math 244 Exam 2, Spring 2023

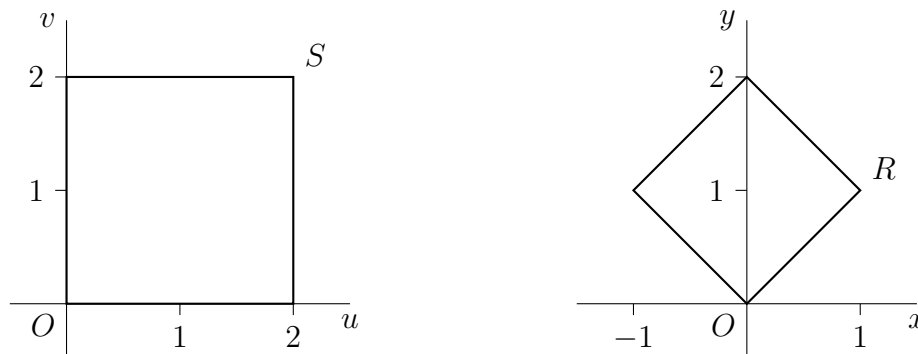
Name:

Question	Points	Score
1	11	
2	13	
3	9	
4	8	
Total:	41	

- You have 50 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 3x5 notecard with notes.
- No electronic devices are authorized with the exception of a scientific calculator.
- You do not need to simplify your answers.
- The back side of each page can be used as scratch paper.
- Good luck!

Homework	
Exam 1	
Exam 2	
Total	

1. Let R and S be the square regions in the xy -plane and uv -plane, respectively, picture below.



- (a) (2 points) Consider the transformation from the xy -plane to the uv -plane given by

$$u = y + x \quad \text{and} \quad v = y - x.$$

This mapping is an invertible linear transformation which means that it maps parallelograms in the xy -plane to parallelograms in the uv -plane. In particular, this transformation maps *corners* of parallelograms to *corners* of parallelograms. Use this fact to show that the transformation maps the region R onto the region S .

- (b) (3 points) Find the inverse transformation by solving for x and y in terms of u and v .

(c) (3 points) Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation you found in part (b).

(d) (3 points) Use parts (a) through (c) to perform a change of variables on the integral

$$\iint_R (y^2 - x^2)^3 dA,$$

You do not need to evaluate the integral.

2. (13 points) Evaluate the following integral by switching to spherical coordinates

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} e^{(x^2+y^2+z^2)^{3/2}} dz dy dx.$$

3. (9 points) Evaluate the triple integral $\int_0^\pi \int_0^\pi \int_0^\pi \sin(x + y + z) \, dx dy dz$.

4. Let E be the solid above the xy -plane bounded by the sphere $x^2 + y^2 + z^2 = 1$, the cylinder $x^2 + y^2 = 1$, and the plane $z = 1$. Let $\rho(x, y, z) = x^2 + y^2$ be the density function of E .
- (a) (2 points) Sketch the solid.
 - (b) (6 points) Set up (but do not evaluate) the mass integral in cylindrical coordinates.
 - (c) (5 points (bonus)) Set up (but do not evaluate) the mass integral in spherical coordinates.