Math 244 Exam 3, Spring 2023

Name:

Question	Points	Score
1	6	
2	7	
3	8	
4	10	
5	8	
6	0	
Total:	39	

- You have 50 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may use a 3x5 notecard with notes.
- No electronic devices are authorized with the exception of a scientific calculator.
- You do not need to simplify your answers.
- The back side of each page can be used as scratch paper.
- Good luck!

Homework	
Exam 1	
Exam 2	
Exam 3	
Total	

1. (6 points) The vector field $\mathbf{F} = 2x\sin(y)\mathbf{i} + x^2\cos(y)\mathbf{j}$ is conservative, find a potential function for $\mathbf{F}$.

2. (7 points) Evaluate

$$\int_C \nabla(\arctan(xyz)) \cdot d\mathbf{r}$$

where C is the path of straight line segments from $(1, 1, 1)$ to $(1, 1, 0)$ to $(1, 0, 0)$ to $(0, 0, 0)$.

3. (8 points) Evaluate

$$\oint_C (x^2y^3 + 2y)dx + x^3y^2dy$$

where C is the boundary of the rectangle $[-1, 1] \times [-1, 1]$ oriented counterclockwise.

4. Let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle y\cos z, x\sin z, xy\sin(z^2)\rangle$.

   (a) (4 points) Determine if $\mathbf{F}$ is conservative.

   (b) (6 points) Find the work done by $\mathbf{F}$ on a particle moving along the path $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$, $-1 \le t \le 1$.

5. Let C be the semicircular arc of $y^2 + z^2 = 1$ that lies above the xy-plane.

   (a) (3 points) Find a parametrization of C.

   (b) (5 points) Evaluate $\int_C e^y \, dy$.

   (c) (1 point (bonus)) Sketch what the integral in part (b) represents.

6. (5 points (bonus)) Let $\mathbf{F}$ be a vector field. A vector field $\mathbf{G}$ is a *vector potential* for $\mathbf{F}$ if $\text{curl}(\mathbf{G}) = \mathbf{F}$. Find a vector potential for $\mathbf{F_1} = \langle 2, 3, 4 \rangle$.