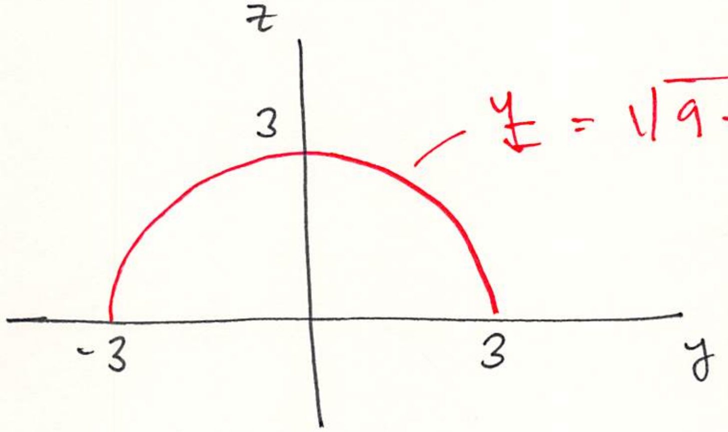
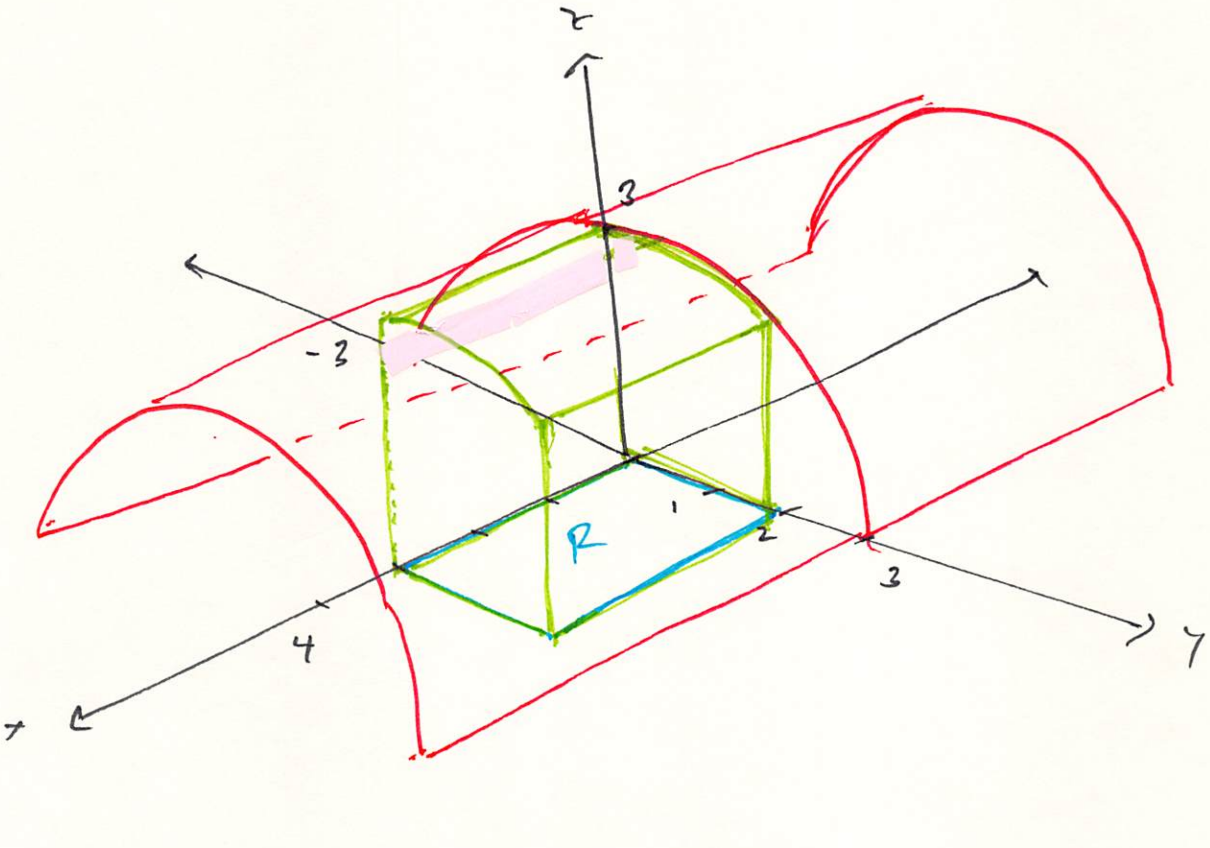
#15.1.12 $\iint_R \sqrt{9-y^2}\, dA$, $R = [0,4] \times [0,2]$

$z = \sqrt{9-y^2}$ is a cylinder (x is free)



Slide this image up and down the x-axis



15.1.18

$$\int_0^{\pi/6}\int_0^{\pi/2} (\sin x + \sin y)\, dy\, dx$$

$$= \int_0^{\pi/6} \left[y \cdot \sin x - \cos y \right]_0^{\pi/2} dx$$

$$= \int_0^{\pi/6} \frac{\pi}{2} \cdot \sin x - \cos\left(\frac{\pi}{2}\right) - \left(0 \cdot \sin x - \cos(0)\right) dx$$

$$= \int_0^{\pi/6} \frac{\pi}{2} \cdot \sin x + 1\, dx$$

$$= -\frac{\pi}{2} \cdot \cos x + x \Big|_0^{\pi/6}$$

$$= -\frac{\pi}{2} \cdot \cos\left(\frac{\pi}{6}\right) + \frac{\pi}{6} - \left(-\frac{\pi}{2} \cdot \cos(0) + 0\right)$$

$$= -\frac{\pi}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\pi}{6} + \frac{\pi}{2}$$

$$= \frac{\pi}{12}\left(8 - 3\sqrt{3}\right)$$

15.1.32

$$\int_0^1 \int_0^1 \frac{x}{1+xy}\, dy\, dx$$

$u(y) = 1+xy \qquad u(0) = 1 + x\cdot 0 = 1$

$du = x\,dy \qquad u(1) = 1+x\cdot 1 = 1+x$

$$= \int_0^1 \int_1^{1+x} \frac{1}{u}\, du\, dx$$

$$= \int_0^1 \ln|u| \Big]_1^{1+x} dx$$

$$= \int_0^1 \ln|1+x| - \underbrace{\ln|1|}_{0}\, dx$$

since $0 \le x \le 1$, then $|1+x| = 1+x$

$$= \int_0^1 \ln(1+x)\, dx$$

parts

$\longrightarrow$

cont.

$u = \ln(1+x)$ $\qquad dv = dx$

$du = \frac{1}{1+x} dx$ $\qquad v = x$

$$= x \cdot \ln(1+x) \Big]_0^1 - \int_0^1 \frac{x}{1+x} dx$$

$$= \ln 2 - \int_0^1 \frac{x+1-1}{x+1} dx$$

$$= \ln 2 - \int_0^1 1 - \frac{1}{x+1} dx$$

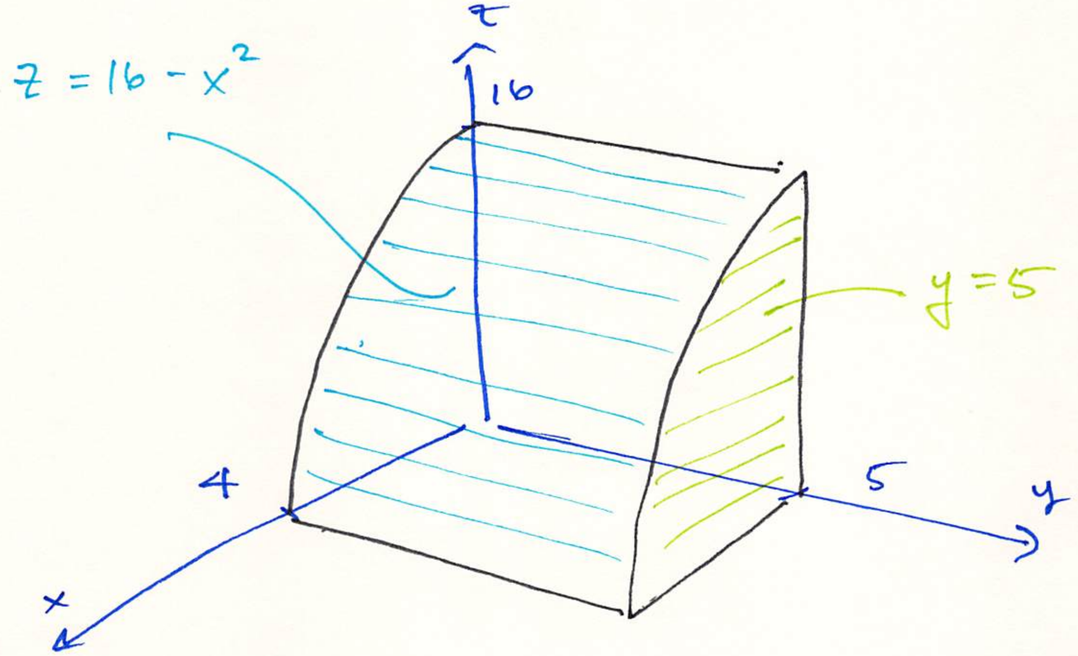
$$= \ln 2 - \Big[x - \ln|x+1| \Big]_0^1$$

$$= \ln 2 - \big[1 - \ln 2 - (0-0) \big]$$

$$= 2 \ln 2 - 1$$

$$= \ln 4 - 1$$

15.1.42 Set-up



$$V = \int_0^4 \int_0^5 16 - x^2 \, dy\, dx$$