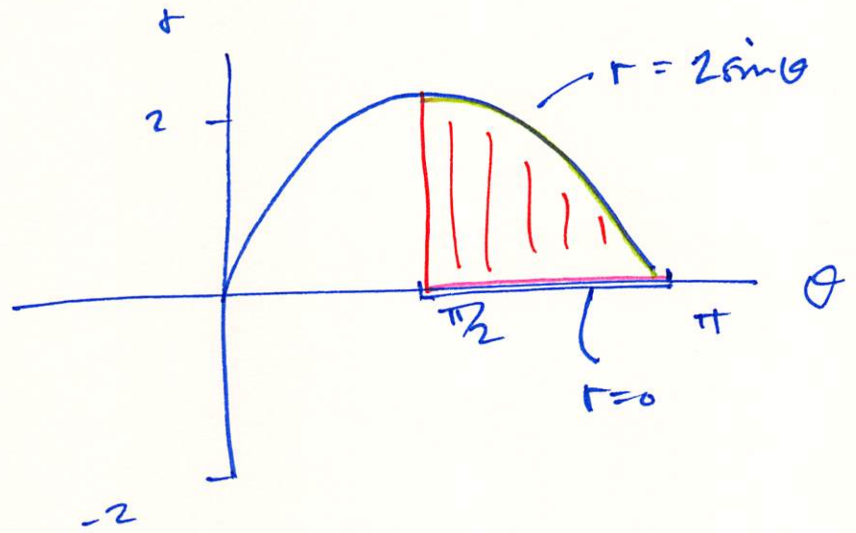
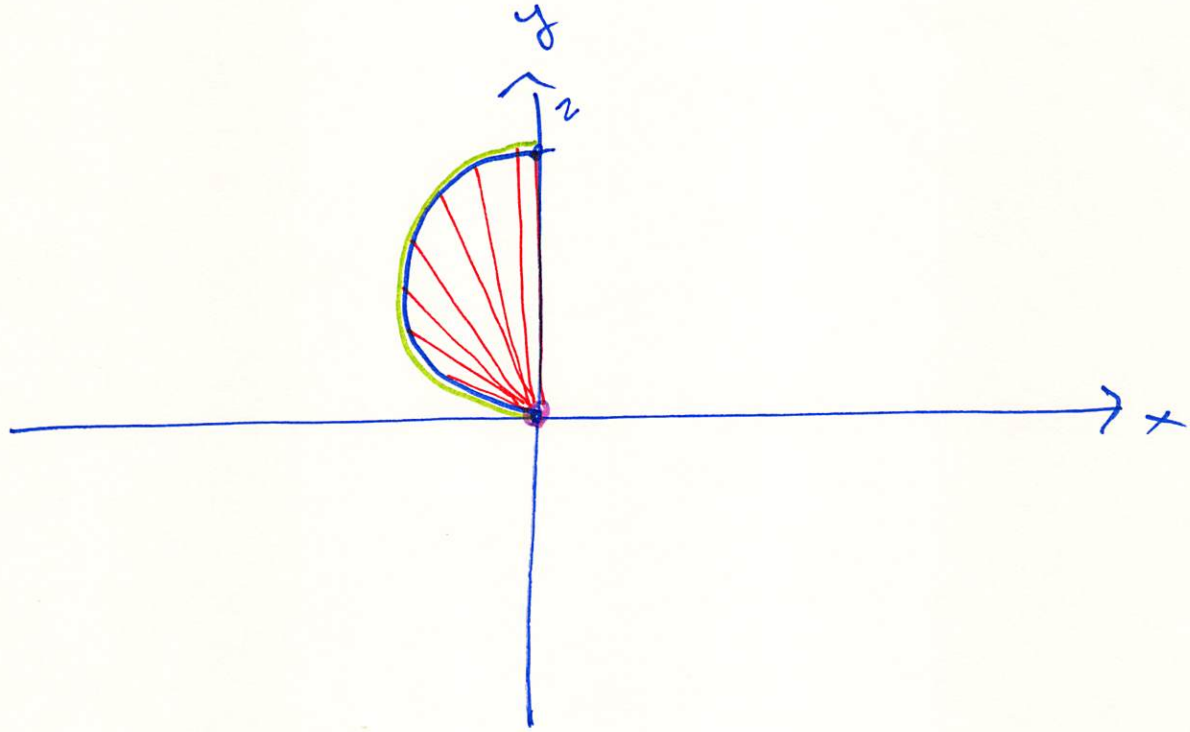
#15.3.6 $\displaystyle\int_{\pi/2}^{\pi}\int_{0}^{2\sin\theta} r\,dr\,d\theta$

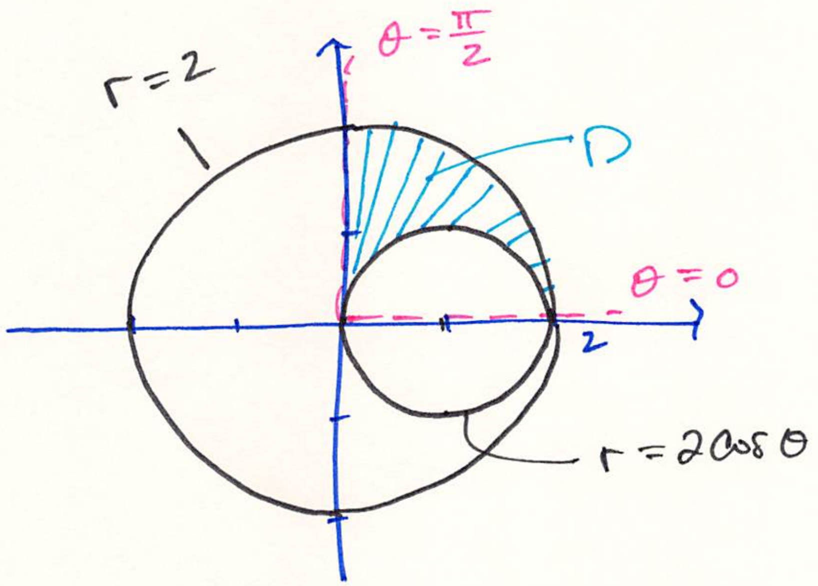




#15.3.14

$$x^2+y^2=2x \iff x^2-2x+y^2=0$$
$$\iff (x-1)^2+y^2=1$$

in polar:
$$r^2=2r\cos\theta$$
$$r=2\cos\theta$$



$$\iint_D x\,dA = \int_0^{\pi/2}\int_{2\cos\theta}^{2} r\cos\theta\cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \cos\theta\cdot \frac{r^3}{3}\Big|_{2\cos\theta}^{2}\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/2} 8\cos\theta - 8\cos^4\theta\,d\theta$$

$$\cos^4\theta = \left(\frac{1+\cos 2\theta}{2}\right)^2$$

$$= \frac{1}{4}\left(1 + 2\cos 2\theta + \cos^2 2\theta\right)$$

$$= \frac{1}{4}\left(1 + 2\cos 2\theta + \frac{1}{2}\left(1 + \cos 4\theta\right)\right)$$

$$= \frac{1}{4}\left(\frac{3}{2} + 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right)$$

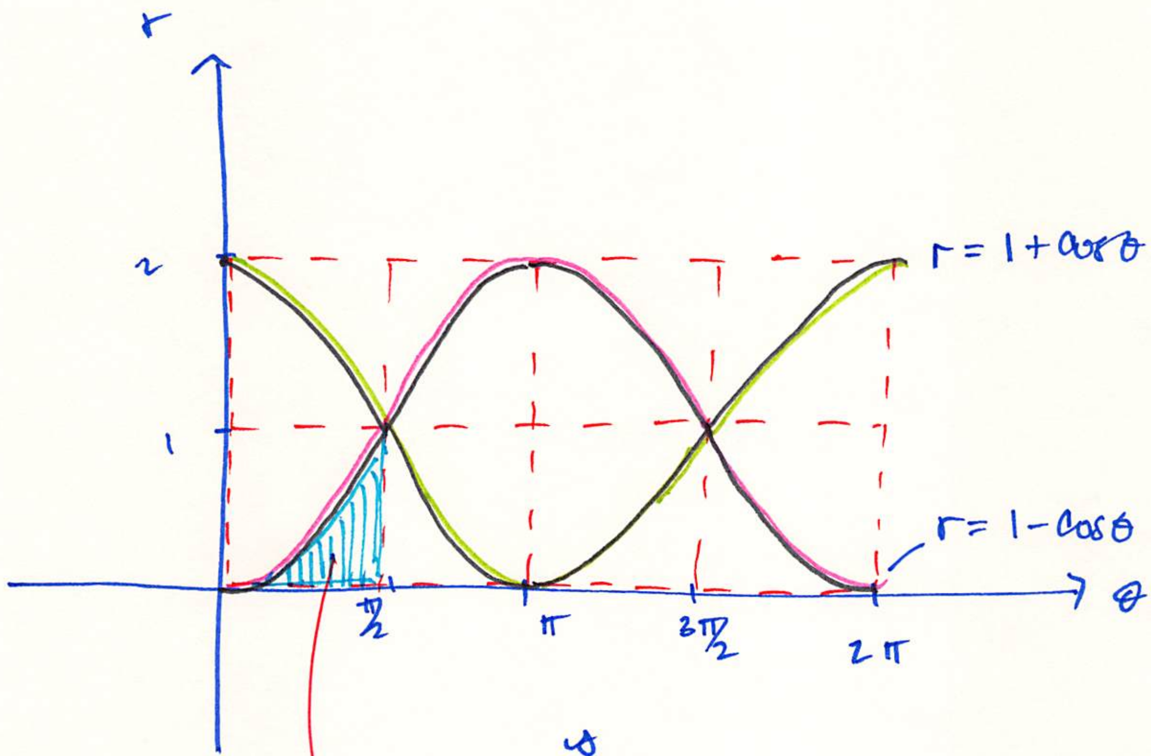
$$\text{integral} = \frac{8}{3}\int_0^{\pi/2} \cos\theta - \frac{1}{4}\left(\frac{3}{2} + 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right) d\theta$$

$$= \frac{1}{3}\left[8\sin\theta - 3\theta + 4\cdot\frac{1}{2}\cancel{\sin 2\theta} + \frac{1}{4}\cancel{\sin 4\theta}\right]_0^{\pi/2}$$

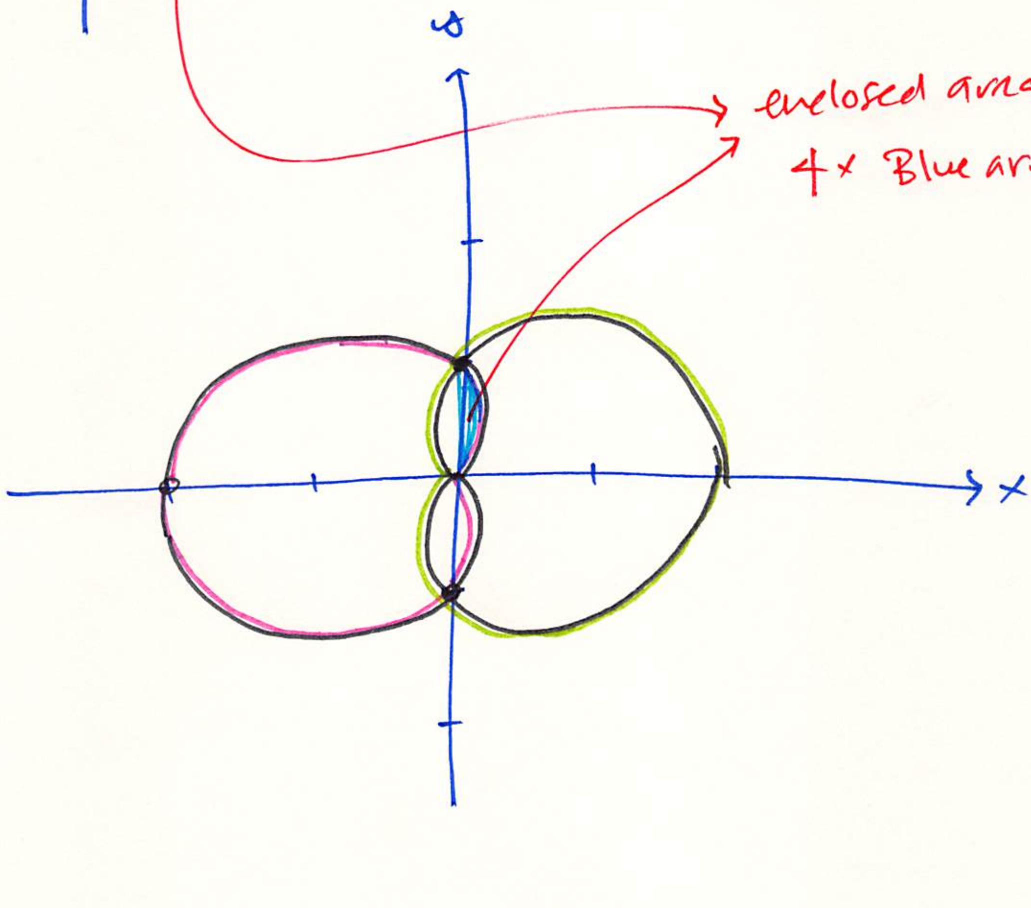
$$= \frac{1}{3}\left[8 - \frac{3\cdot\pi}{2}\right]$$

$$= \frac{1}{6}\left(16 - 3\pi\right) \quad \text{or} \quad \frac{8}{3} - \frac{\pi}{2}$$

#15.3.16



enclosed area is
4× Blue area.



$$A = 4 \cdot \int_0^{\pi/2} \int_0^{1+\cos\theta} r \, dr \, d\theta$$

$$= 4 \cdot \int_0^{\pi/2} \frac{1}{2} r^2 \Big|_0^{1+\cos\theta} d\theta$$

$$= 2 \int_0^{\pi/2} (1+\cos\theta)^2 \, d\theta$$

$$= 2 \int_0^{\pi/2} 1 + 2\cos\theta + \cos^2\theta \, d\theta$$

$$= 2 \int_0^{\pi/2} 1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta) \, d\theta$$

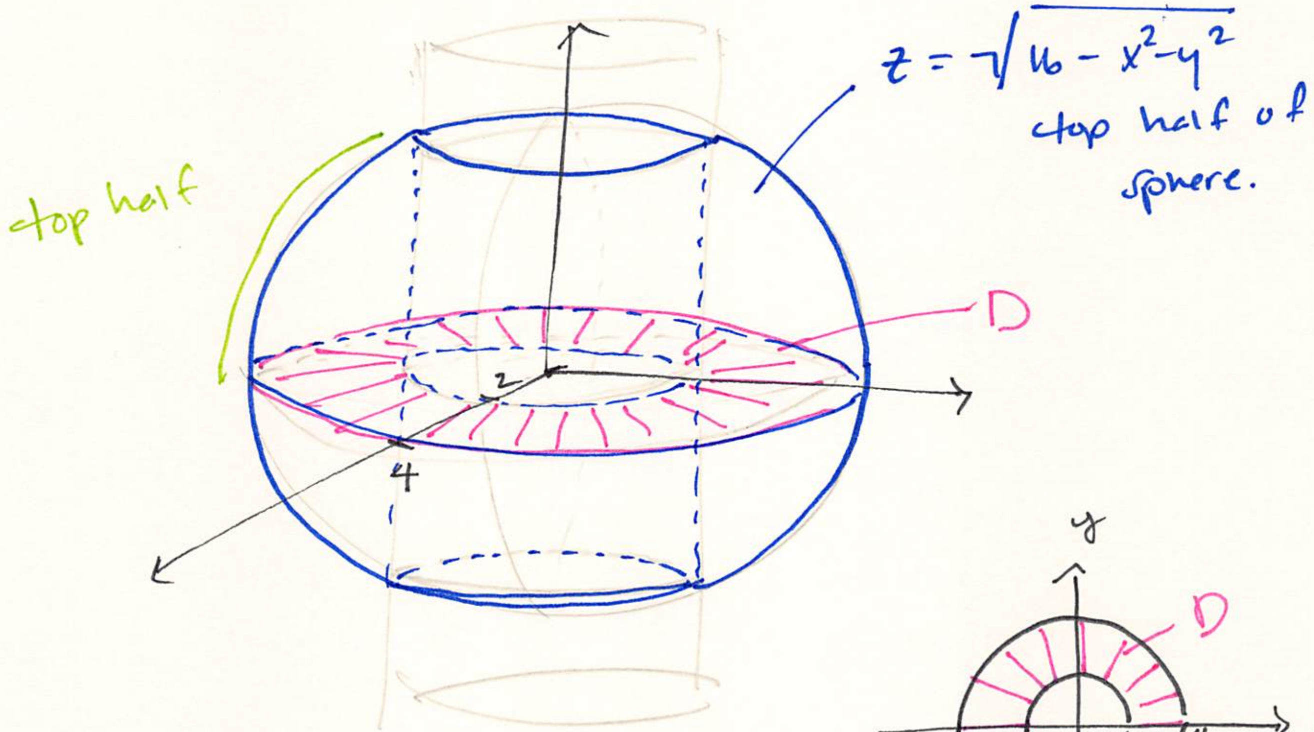
$$= \int_0^{\pi/2} 3 + 4\cos\theta + \cos 2\theta \, d\theta$$

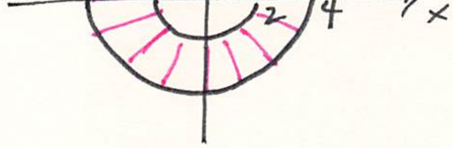
$$= \Big[3\theta + 4\sin\theta + \frac{1}{2} \cancel{\sin 2\theta} \Big]_0^{\pi/2}$$

$$= \frac{3\pi}{2} + 4$$

15.3.22 Inside sphere $x^2+y^2+z^2=16$

& outside cylinder $x^2+y^2=4$





D in polar: $2 \leq r \leq 4$

$0 \leq \theta \leq 2\pi$

$V = 2 \times$ Volume of top half

$= 2 \cdot \iint_D \sqrt{16-x^2-y^2}\, dA$

$= 2 \cdot \int_0^{2\pi} \int_2^4 \sqrt{16-r^2}\; r\,dr\,d\theta$

$\longrightarrow$

$$= 4\pi \cdot \int_2^4 \sqrt{16-r^2}\, r\,dr$$

$u = 16 - r^2 \qquad u(2) = 16 - 4 = 12$

$du = -2r\,dr \qquad u(4) = 0$

$$= 4\pi \cdot \int_{12}^{0} u^{1/2} \left(-\frac{1}{2}\,du\right)$$

$$= 4\pi \left[\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}\, u^{3/2} \right]_0^{12}$$

$$= \frac{4\pi}{3} \cdot 12^{3/2}$$

$$= \frac{32\pi}{3} \cdot 3^{3/2}$$