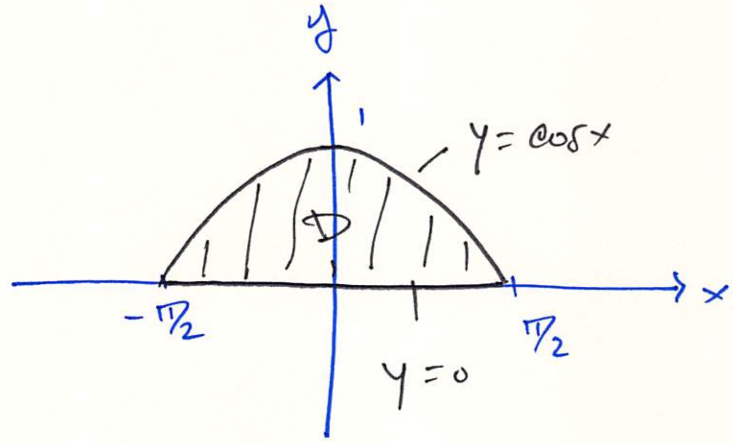
\# 15.4.10

Density

$\rho = y.$



$$m = \iint_D \rho(x,y)\,dA = \iint_D y\,dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} y\,dy\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{y^2}{2}\Big|_0^{\cos x} dx$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$$

$$= \frac{1}{4}\left[x + \frac{1}{2}\cancel{\sin 2x}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{\pi}{4}$$

$$M_x = \iint_D y\,\rho(x,y)\,dA = \iint_D y^2\,dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} y^2\,dy\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{y^3}{3}\Big|_0^{\cos x}\,dx$$

even

$$= \frac{1}{3}\int_{-\pi/2}^{\pi/2} \cos^3 x\,dx$$

$$= \frac{2}{3}\int_0^{\pi/2} (1-\sin^2 x)\cos x\,dx \qquad \begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}$$

$$= \frac{2}{3}\int_0^1 1-u^2\,du$$

$$= \frac{2}{3}\left[u - \frac{u^3}{3}\right]_0^1$$

$$= \frac{2}{3}\left(\frac{2}{3} - 0\right) = \frac{4}{9}.$$

$$M_y = \iint_D x \, dA = \iint_D xy \, dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} xy \, dy \, dx$$

$$= \int_{-\pi/2}^{\pi/2} x \cdot \frac{y^2}{2} \Big|_0^{\cos x} dx$$

odd

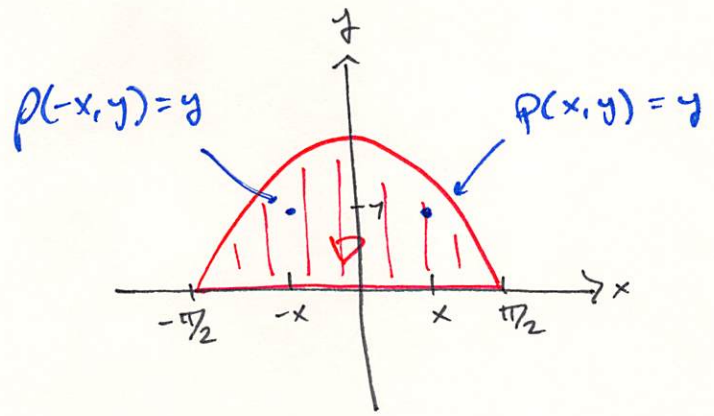
$$= \frac{1}{2} \int_{-\pi/2}^{\pi/2} x \cdot \cos^2 x \, dx$$

$$= 0.$$

$$\bar{x} = \frac{M_y}{m} = 0$$

$$\bar{y} = \frac{M_x}{m} = \frac{4/9}{\pi/4} = \frac{16}{9\pi}$$

Alternative argument that $\bar{x} = 0$.



The density function is symmetric about the y-axis. Also, the lamina is symmetric about the y-axis

$$\Longrightarrow \bar{x} = 0.$$

15.4.29

$$f(x,y) = \begin{cases} 0.1\, e^{-(0.5x + 0.2y)} & x \geq 0, y \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

(a) The natural exponential is a positive function so $f(x,y)$ is positive.

$$\iint_{\mathbb{R}^2} f(x,y)\, dA = \int_0^\infty \int_0^\infty 0.1 e^{-(0.5x + 0.2y)}\, dy\, dx$$

$$= 0.1 \int_0^\infty e^{-0.5x}\, dx \cdot \int_0^\infty e^{-0.2y}\, dy$$

$$= 0.1 \left[\lim_{t \to \infty} \frac{e^{-0.5x}}{-0.5} \Big|_0^t \right] \cdot \left[\lim_{t \to \infty} \frac{e^{-0.2y}}{-0.2} \Big|_0^t \right]$$

$$= 0.1 \cdot \left[\frac{1}{0.5} \right] \cdot \left[\frac{1}{0.2} \right]$$

$$= 1$$

$$(bi)\quad P(Y \geq 1) = \iint_D f(x,y)\, dA$$

$$= \int_0^\infty \int_1^\infty 0.1 e^{-(0.5x + 0.2y)}\, dy\, dx$$

$$= 0.1 \int_0^\infty e^{-0.5x}\, dx \cdot \int_1^\infty e^{-0.2y}\, dy$$

$$= 0.1 \left[\lim_{t\to\infty} \frac{e^{-0.5x}}{-0.5} \Big|_0^t \right] \cdot \left[\lim_{t\to\infty} \frac{e^{-0.2y}}{-0.2} \Big|_1^t \right]$$

$$= 0.1 \left[\frac{1}{0.5} \right] \left[\frac{e^{-0.2}}{0.2} \right]$$

$$= e^{-0.2}.$$

$$\mu_1 = \iint_{\mathbb{R}^2} x\, f(x,y)\, dA = \int_0^\infty \int_0^\infty x \cdot 0.1 e^{-(0.5x + 0.2y)}\, dy\, dx$$

$$= 0.1 \cdot \int_0^\infty x e^{-0.5x}\, dx \cdot \int_0^\infty e^{-0.2y}\, dy$$

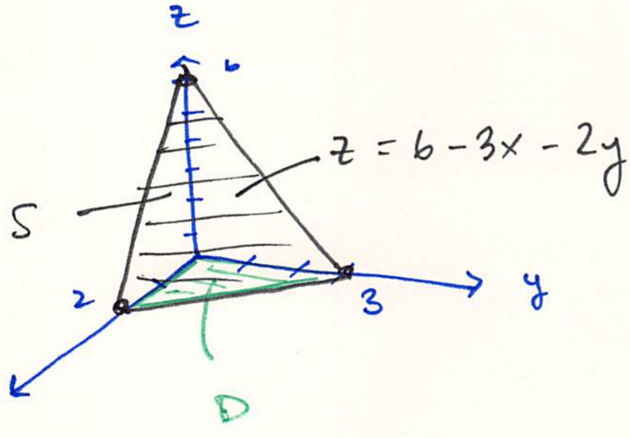
$$\begin{array}{ccc} x & & e^{-0.5x} \\ & + & \\ 1 & & \dfrac{e^{-0.5x}}{-0.5} \\ & - & \\ 0 & & \dfrac{e^{-0.5x}}{0.25} \end{array}$$

$$= 0.1 \cdot \left[\lim_{t \to \infty} \left(- \frac{x e^{-0.5x}}{0.5} - \frac{e^{-0.5x}}{0.25} \right)\Bigg|_0^\infty \right] \cdot \frac{1}{0.2}$$

$$= 0.1 \left[\frac{1}{0.25} \right] \left[\frac{1}{0.2} \right]$$

$$= 0.1 \cdot \frac{1}{0.5} = \frac{1}{10} \cdot \frac{1}{5/10} = \frac{1}{5}$$

15.5.3



$$\frac{\partial z}{\partial x} = -3, \quad \frac{\partial z}{\partial y} = -2$$

$$\Longrightarrow \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1+9+4} = \sqrt{14}$$

$$A(S) = \iint_D \sqrt{1 + \left(\partial z/\partial x\right)^2 + \left(\partial z/\partial y\right)^2}\, dA$$

$$= \iint_D \sqrt{14}\, dA$$

$$= \sqrt{14} \iint_D 1 \cdot dA$$

$$= \sqrt{14} \cdot A(D) = \sqrt{14} \cdot \frac{1}{2} \cdot 2 \cdot 3$$

$$= 3\sqrt{14}$$