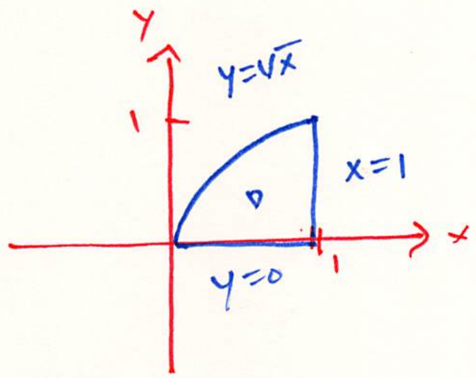
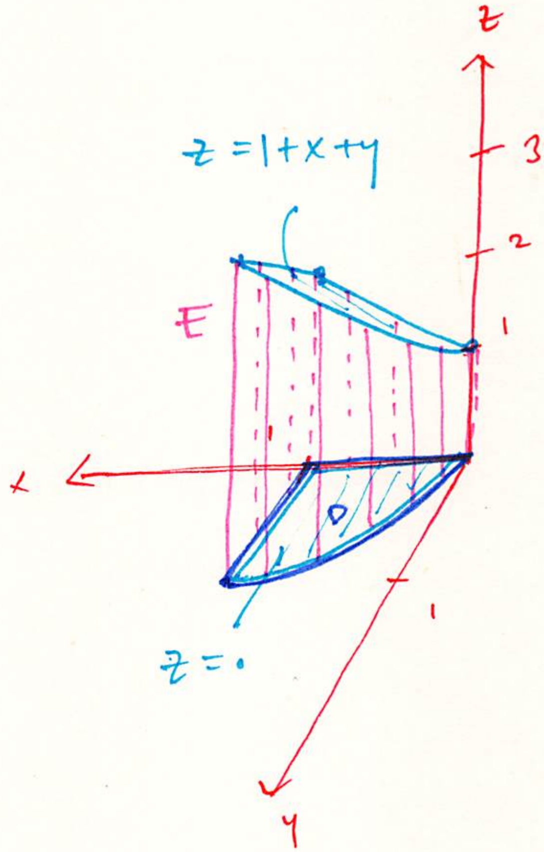
15.6.13





$$\iiint_E 6xy\,dV$$

$$= \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy \; dz\,dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} 6xyz \Big]_0^{1+x+y} dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} 6xy\left(1+x+y-0\right) dy\,dx$$

$$= 6\int_0^1 \int_0^{\sqrt{x}} \left(x+x^2\right)y + xy^2 \; dy\,dx$$

$\longrightarrow$

$$= 6\int_0^1 (x+x^2)\frac{y^2}{2} + x\frac{y^3}{3}\Big]_0^{\sqrt{x}} dx$$

$$= \int_0^1 3(x+x^2)x + 2x\cdot x^{3/2}\, dx$$

$$= \int_0^1 3x^2 + 3x^3 + 2x^{5/2}\, dx$$

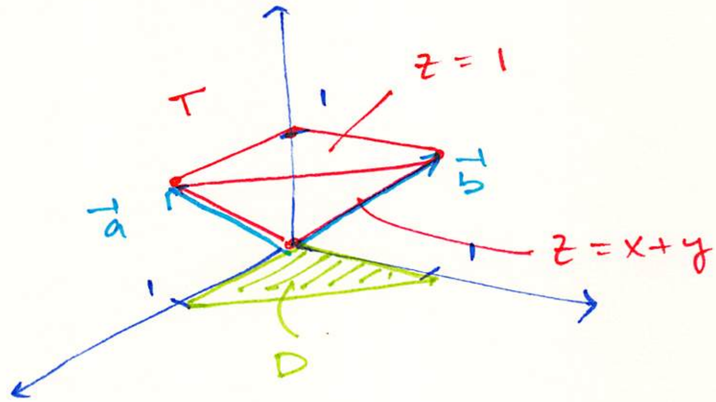
$$= x^3 + \frac{3}{4}x^4 + 2\cdot\frac{2}{7}x^{7/2}\Big|_0^1$$

$$= 1 + \frac{3}{4} + \frac{4}{7}$$

$$= \frac{28+21+16}{28}$$

$$= \frac{65}{28}$$

15.6.16



$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - 1(\hat{j} - \hat{k}) + 0$$

$$= -\hat{i} - \hat{j} + \hat{k} \quad \text{normal vector}$$

$\Longrightarrow$ Plane spanned by $\vec{a}$ and $\vec{b}$ is

$$-1(x-0) - 1(y-0) + 1(z-0) = 0$$

$$-x - y + z = 0$$

$$z = x + y$$

$$\iiint_T xz\, dV = \int_0^1 \int_0^{1-x} \int_{x+y}^{1} xz \, dz\, dy\, dx$$

$$= \int_0^1 \int_0^{1-x} x \, \frac{z^2}{2} \Big|_{x+y}^{1} \, dy \, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^{1-x} x \left(1 - (x+y)^2 \right) dy \, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^{1-x} x \left(1 - x^2 - 2xy - y^2 \right) dy \, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^{1-x} (x - x^3) - 2x^2 y - x y^2 \, dy \, dx$$

$$= \frac{1}{2} \int_0^1 (x - x^3) y - x^2 y^2 - \frac{x}{3} y^3 \Big]_0^{1-x} dx$$

$$= \frac{1}{2} \int_0^1 (x + x^2)(1-x)^2 - x^2 (1-x)^2 - \frac{x}{3} (1-x)^3 \, dx$$

$$= \frac{1}{6} \int_0^1 (1-x)^2 \left[3x + \cancel{3x^2} - \cancel{3x^2} - x(1-x) \right] dx$$

$$x^2 - x + 3x$$

$$x^2 + 2x$$

$$(1-x)^2\,(x^2+2x)$$

$$= (1-2x+x^2)(x^2+2x)$$

$$= x^2 - \cancel{2x^3} + \underline{x^4} + 2x - 4x^2 + \cancel{2x^3}$$

$$= x^4 - 3x^2 + 2x$$

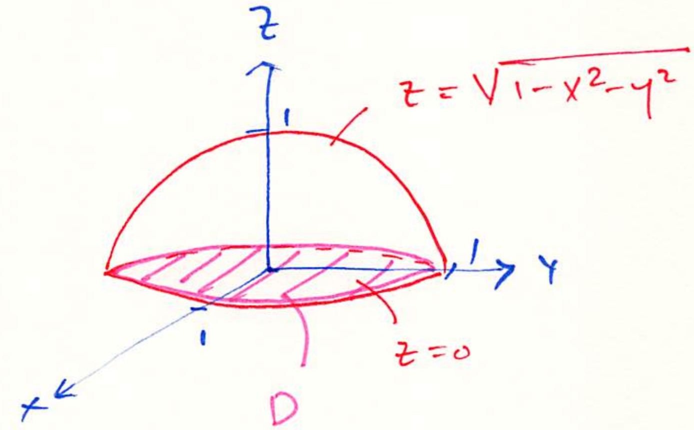
$$\text{Integral} = \frac{1}{6}\int_0^1 x^4 - 3x^2 + 2x \, dx$$

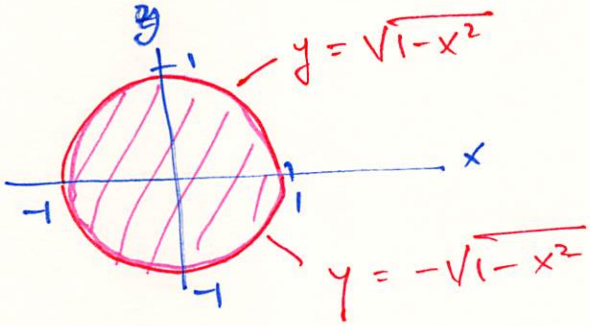
$$= \frac{1}{6}\left[\frac{x^5}{5} - x^3 + x^2\right]_0^1$$

$$= \frac{1}{6}\left(\frac{1}{5} - 1 + 1\right)$$

$$= \frac{1}{30}$$

15.6.48





$$\rho(x,y,z)=\sqrt{x^2+y^2+z^2}$$

$$m=\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}}\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$$

$$\bar{x}=\frac{1}{m}\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}}x\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$$

$$\bar{y}=\frac{1}{m}\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}}y\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$$

$$\bar{z} = \frac{1}{m} \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

$$I_z = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} (x^2+y^2)\sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$