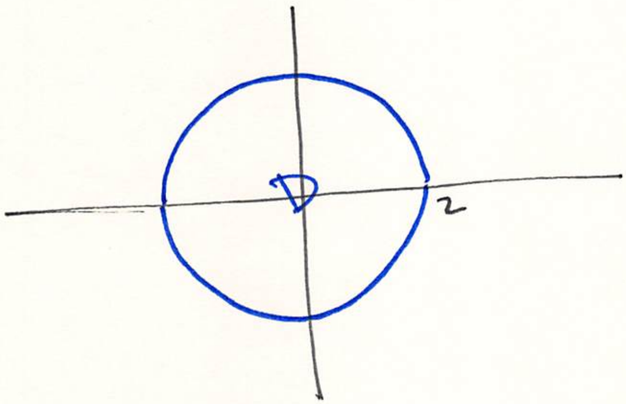
#15.7.16 $\underbrace{\int_0^2 \int_0^{2\pi}}_{D} \int_0^r r \, dz \, d\theta \, dr$

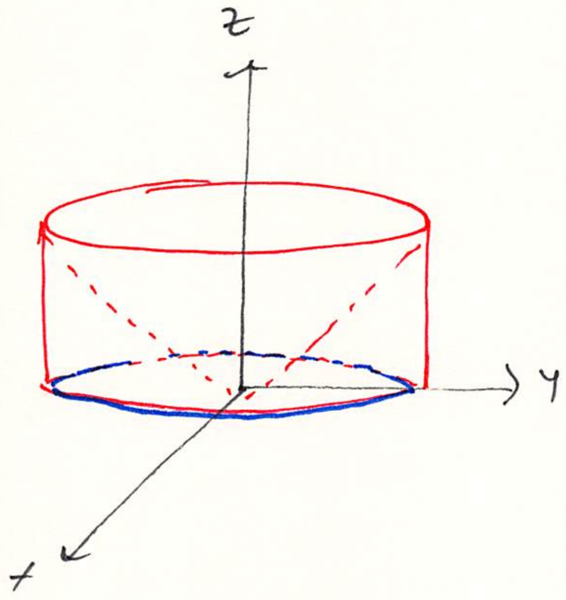


$0 \leq z \leq r$

$0 \leq z \leq \sqrt{x^2+y^2}$

cone

plane



15.7.20

$$\iiint_E (x-y)\,dV = \int_0^{2\pi}\int_1^4\int_0^{r\sin\theta+4} (r\cos\theta - r\sin\theta)\, r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^4 r^2(\cos\theta - \sin\theta)\, z \Big|_0^{r\sin\theta+4} dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^4 r^2(\cos\theta - \sin\theta)(r\sin\theta + 4)\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^4 r^2\left(r\sin\cancel{\theta\cos}\theta + 4\cancel{\cos}\theta - r\sin^2\theta - \cancel{4\sin}\theta\right)dr\,d\theta$$

0 by symmetry

$$= \int_0^{2\pi}\int_1^4 -r^3\sin^2\theta\,dr\,d\theta$$

$$= - \left[\frac{r^4}{4} \right]_1^4 \left[\frac{1}{2} \left(\theta - \frac{1}{2} \cancel{\sin} 2\theta \right) \right]_0^{2\pi}$$

$$= - \left[4^3 - \frac{1}{4} \right] \left[\pi \right]$$

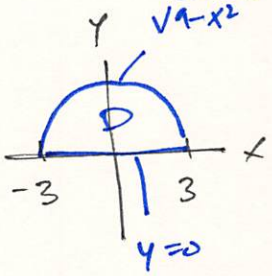
$$= - \frac{\pi}{4} \left(4^4 - 1 \right)$$

$$= - \frac{\pi}{4} \left(255 \right)$$

$$= - \frac{255\pi}{4}$$

#15.7.30

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2}\sqrt{x^2+y^2}\;dz\,dy\,dx$$



$$= \int_{0}^{\pi}\int_{0}^{3}\int_{0}^{9-r^2}\sqrt{r^2}\;r\,dz\,dr\,d\theta$$

$$= \int_{0}^{\pi}\int_{0}^{3} r^2 z\Big]_{0}^{9-r^2}\,dr\,d\theta$$

$$= \int_{0}^{\pi}\int_{0}^{3} r^2(9-r^2)\,dr\,d\theta$$

$$= \int_{0}^{\pi}\int_{0}^{3} 9r^2 - r^4\,dr\,d\theta$$

$$= \pi\cdot\left[3r^3 - \frac{r^5}{5}\right]_{0}^{3}$$

$$= \pi\left(81 - \frac{243}{5}\right) = \frac{162\pi}{5}$$