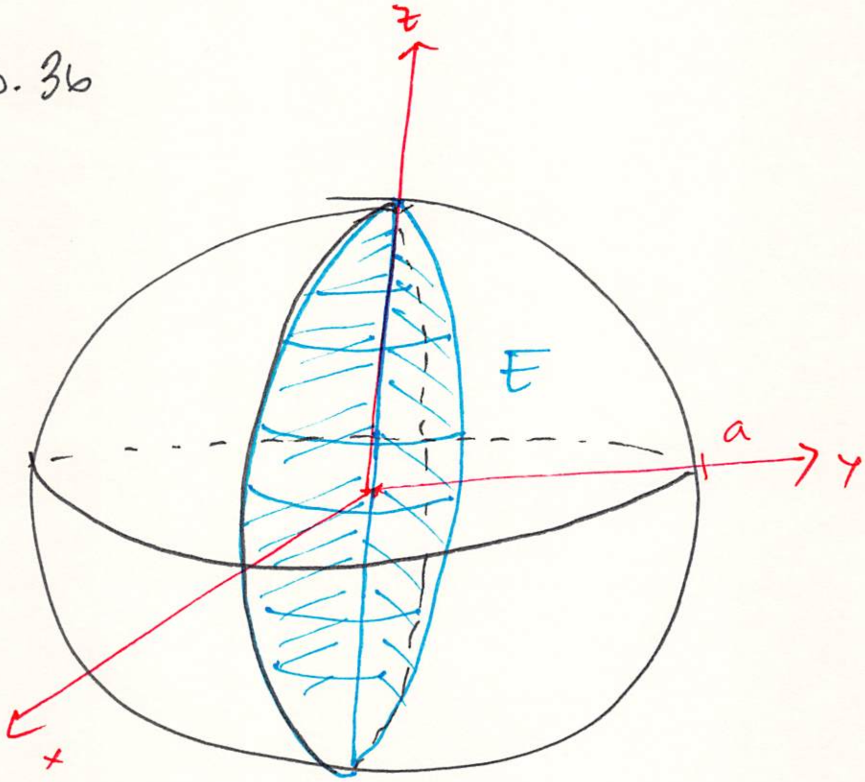
15.8.20

$$\int_0^{\pi/2}\int_{\pi/2}^{2\pi}\int_1^2 f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)\ \rho^2\sin\phi\, d\rho\, d\theta\, d\phi$$

\# 15.8.36



$$V = \iiint_E 1 \, dV = \int_0^{\pi/6} \int_0^{\pi} \int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \frac{\pi}{6} \underbrace{\left[-\cos\phi\right]_0^{\pi}}_{2} \left[\rho^3/3\right]_0^{a}$$

$$= \frac{a^3\pi}{9} \quad \text{or} \quad \frac{\pi a^3}{9}$$

15.9.24

$$\iint_R (x+y)e^{x^2-y^2}\,dA = \iint_R \overbrace{(x+y)}^{u} e^{\overbrace{(x+y)}^{u}\overbrace{(x-y)}^{v}}\,dA$$

Let $u = x+y$, $v = x-y$ } this is T^{-1}

Solve for x and y to find T.

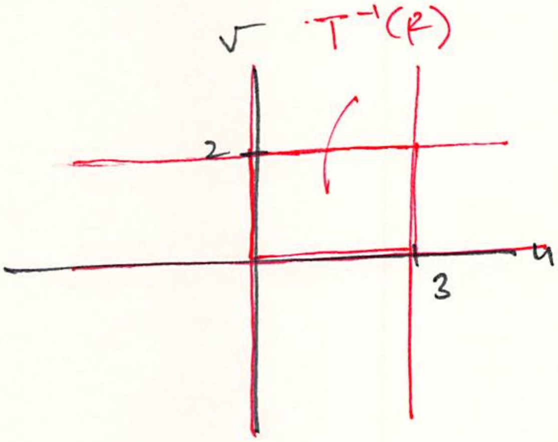
$u+v = 2x \Rightarrow x = \frac{1}{2}(u+v)$

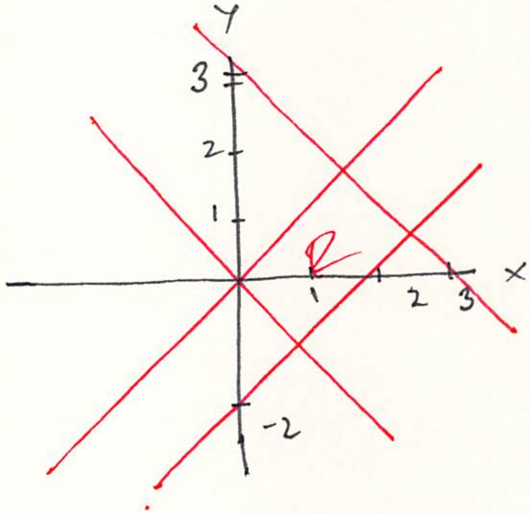
$u-v = +2y \Rightarrow y = \frac{1}{2}(u-v)$

$$\Rightarrow \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

R is the region enclosed by the lines

$$\begin{aligned} x-y&=0 \\ x-y&=2 \\ x+y&=0 \\ x+y&=3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} v&=0 \\ v&=2 \\ u&=0 \\ u&=3 \end{aligned}$$





$$\therefore \iint_R (x+y)e^{x^2-y^2}\,dA = \iint_{T^{-1}(R)} u e^{uv} \left|-\tfrac{1}{2}\right| du\,dv$$

$$= \frac{1}{2}\int_0^3 \int_0^2 u e^{uv} \, dv \, du$$

$$s = uv \qquad s(0) = 0$$

$$ds = u \, dv \qquad s(2) = 2u$$

$$= \frac{1}{2}\int_0^3 \int_0^{2u} e^{s} \, ds \, du$$

$$= \frac{1}{2}\int_0^3 e^{s} \Big|_0^{2u} du$$

$$= \frac{1}{2}\int_0^3 e^{2u} - 1 \, du$$

$$= \frac{1}{2}\left(\frac{1}{2}e^{2u} - u\right)\Big|_0^3$$

$$= \frac{1}{2}\left(\frac{1}{2}e^{6} - 3 - \left(\frac{1}{2} - 0\right)\right)$$

$$= \frac{1}{4}\left(e^{6} - 6 - 1\right) = \frac{1}{4}\left(e^{6} - 7\right)$$