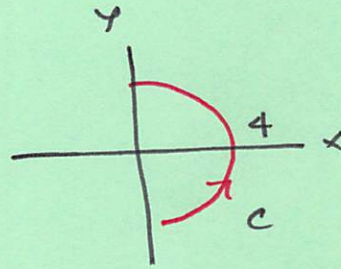


# 16.2.3

$$\int_c xy^4 ds$$



$$\vec{r}(t) = \langle 4 \cos t, 4 \sin t \rangle, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

$$\vec{r}'(t) = \langle -4 \sin t, 4 \cos t \rangle$$

$$\|\vec{r}'(t)\| = \sqrt{16 \sin^2 t + 16 \cos^2 t} = 4$$

$$\int_c xy^4 ds = \int_{-\pi/2}^{\pi/2} (4 \cos t) (4 \sin t)^4 \cdot 4 dt$$

$$u = 4 \sin t \quad u(-\pi/2) = -4$$

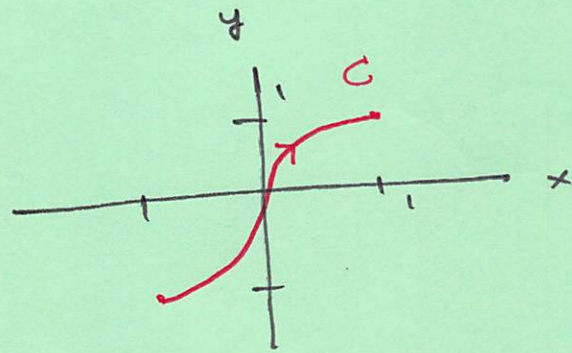
$$du = 4 \cos t dt \quad u(\pi/2) = 4$$

$$= \int_{-4}^4 4u^4 dt = 2 \int_0^4 4u^4 du$$

$$= \frac{8}{5} u^5 \Big|_0^4$$

$$= \frac{8}{5} \cdot 1024$$

$$(b) \int_C e^x dx,$$



$$\vec{r}(t) = \langle t^3, t \rangle, \quad -1 \leq t \leq 1$$

$$\vec{r}'(t) = \langle 3t^2, 1 \rangle$$

$$\|\vec{r}'(t)\| = \sqrt{9t^4 + 1}$$

$$\int_C e^x dx = \int_{-1}^1 e^{t^3} 3t^2 dt$$

$$u = t^3$$

$$du = 3t^2 dt$$

$$u(-1) = -1$$

$$u(1) = 1$$

$$= \int_{-1}^1 e^u du$$

$$= e^u \Big|_{-1}^1$$

$$= e - e^{-1}$$

$$(16) \int_C (y+z) dx + (x+z) dy + (x+y) dz$$

The diagram shows a path  $C$  in 3D space. The path starts at point  $(0,0,0)$  and goes to  $(1,0,1)$  via curve  $C_1$ . From  $(1,0,1)$ , it goes to  $(0,1,2)$  via curve  $C_2$ . A curved arrow labeled  $\sigma$  indicates the direction of the path.

$$C_1: \vec{r}(t) = (1-t)\langle 0,0,0 \rangle + t\langle 1,0,1 \rangle$$

$$= \langle t, 0, t \rangle, \quad 0 \leq t \leq 1$$

$$\vec{r}'(t) = \langle 1, 0, 1 \rangle$$

$$\int_{C_1} \sigma = \int_0^1 (0+t) \cdot 1 + (t+t) \cdot 0 + (t+0) \cdot 1 \, dt$$

$$= \int_0^1 t + 2t + t \, dt$$

$$= 4 \int_0^1 t \, dt$$

$$= 2t^2 \Big|_0^1 = 2.$$

$$\begin{aligned}
 C_2: \vec{r}(t) &= (1-t)\langle 1, 0, 1 \rangle + t\langle 0, 1, 2 \rangle \\
 &= \langle 1-t, 0, 1-t \rangle + \langle 0, t, 2t \rangle \\
 &= \langle 1-t, t, 1+t \rangle, \quad 0 \leq t \leq 1
 \end{aligned}$$

$$\vec{r}'(t) = \langle -1, 1, 1 \rangle$$

$$\int_{C_2} \sigma = \int_0^1 (1+2t)(-1) + (2)(1) + (1)(1) dt$$

$$= \int_0^1 -1 - 2t + 2 + 1 dt$$

$$= \int_0^1 2 - 2t dt$$

$$= 2t - t^2 \Big|_0^1$$

$$= 2 - 1 = 1$$

$$\therefore \int_C \sigma = \int_{C_1} \sigma + \int_{C_2} \sigma = 2 + 1 = 3.$$