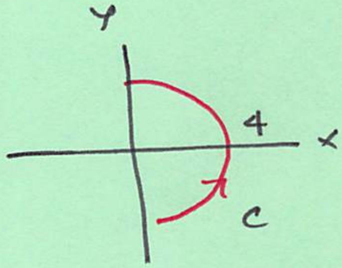
16.2.3 $\int_C xy^4\,ds$



$$\vec{r}(t) = \langle 4\cos t,\ 4\sin t \rangle, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

$$\vec{r}\,'(t) = \langle -4\sin t,\ 4\cos t \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{16\sin^2 t + 16\cos^2 t} = 4$$

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2} (4\cos t)(4\sin t)^4 \cdot 4\,dt$$

$$u = 4\sin t \qquad u(-\pi/2) = -4$$

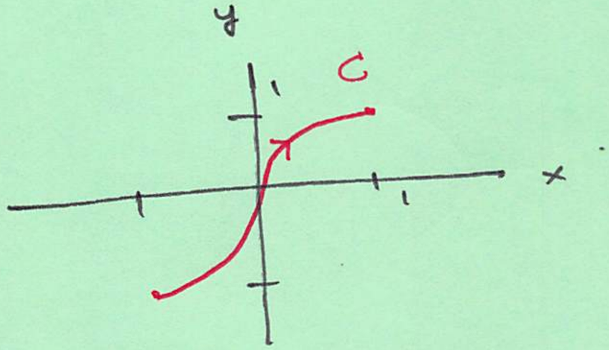
$$du = 4\cos t\,dt \qquad u(\pi/2) = 4$$

$$= \int_{-4}^{4} 4u^4\,dt = 2\int_0^4 4u^4\,du$$

$$= \frac{8}{5}u^5 \Big|_0^4$$

$$= \frac{8}{5} \cdot 1024$$

(b) $\int_C e^x \, dx$,



$\vec{r}(t) = \langle t^3, t \rangle, \quad -1 \le t \le 1$

$\vec{r}\,'(t) = \langle 3t^2, 1 \rangle$

$\|\vec{r}\,'(t)\| = \sqrt{9t^4 + 1}$

$$\int_C e^x \, dx = \int_{-1}^{1} e^{t^3} \, 3t^2 \, dt$$

$u = t^3$

$du = 3t^2 \, dt$

$u(-1) = -1$

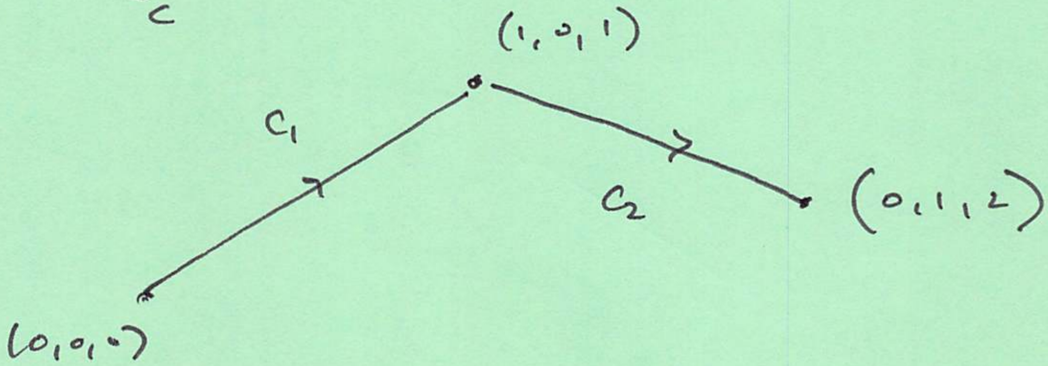
$u(1) = 1$

$$= \int_{-1}^{1} e^u \, du$$

$$= e^u \Big|_{-1}^{1}$$

$$= e - e^{-1}$$

(16) $\int_C \overbrace{(y+z)\,dx + (x+z)\,dy + (x+y)\,dz}^{\sigma}$



$$C_1 : \vec{r}(t) = (1-t)\langle 0,0,0\rangle + t\langle 1,0,1\rangle$$

$$= \langle t,0,t\rangle \,, \quad 0 \le t \le 1$$

$$\vec{r}\,'(t) = \langle 1,0,1\rangle$$

$$\int_{C_1} \sigma = \int_0^1 (0+t)\cdot 1 + (t+t)\cdot 0 + (t+0)\cdot 1 \; dt$$

$$= \int_0^1 t + 2t + t \; dt$$

$$= 4\int_0^1 t\,dt$$

$$= 2t^2 \Big|_0^1 = 2 .$$

C_2: $\vec{r}(t) = (1-t)\langle 1, 0, 1\rangle + t\langle 0, 1, 2\rangle$

$$= \langle 1-t, 0, 1-t\rangle + \langle 0, t, 2t\rangle$$

$$= \langle 1-t, t, 1+t\rangle, \quad 0 \le t \le 1$$

$$\vec{r}\,'(t) = \langle -1, 1, 1\rangle$$

$$\int_{C_2} \sigma = \int_0^1 (1+2t)(-1) + (2)(1) + (1)(1)\, dt$$

$$= \int_0^1 -1 - 2t + 2 + 1\, dt$$

$$= \int_0^1 2 - 2t\, dt$$

$$= 2t - t^2 \Big|_0^1$$

$$= 2 - 1 = 1$$

$$\therefore \int_C \sigma = \int_{C_1} \sigma + \int_{C_2} \sigma = 2 + 1 = 3.$$