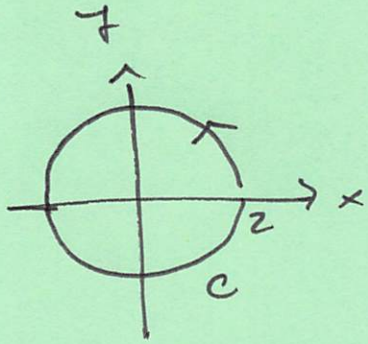
#16.2.32 (a)

$\vec{F} = \langle x^2, xy \rangle$, $\quad C$:



$$\vec{r}(t) = \langle 2\cos t, 2\sin t \rangle$$

$$0 \le t \le 2\pi$$

$$\Longrightarrow \vec{r}\,'(t) = \langle -2\sin t, 2\cos t \rangle$$

$$\vec{F}(\vec{r}(t)) = \vec{F}(2\cos t, 2\sin t)$$

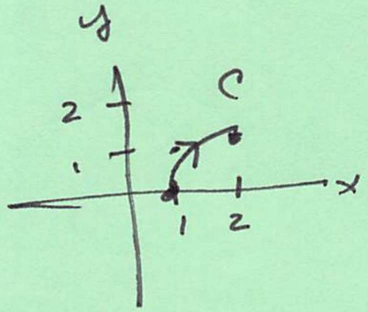
$$= \langle 4\cos^2 t, 4\sin t\cos t \rangle$$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = -8\sin t\cos^2 t + 8\sin t\cos^2 t$$

$$= 0$$

$$\therefore \text{work} = \int_C \vec{F}\cdot d\vec{r} = \int_C 0 = 0.$$

16.2.40 $\vec{F}(x,y) = \langle x^2, ye^x \rangle$



$\vec{r}(t) = \langle 1+t^2, t \rangle, \quad 0 \le t \le 1$

$\vec{r}\,'(t) = \langle 2t, 1 \rangle$

$\vec{F}(\vec{r}(t)) = \vec{F}(1+t^2, t) = \left\langle (1+t^2)^2, te^{1+t^2} \right\rangle$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = (1+t^2)^2 \cdot 2t + te^{1+t^2}$

$$\text{work} = \int_0^1 (1+t^2)^2 2t + te^{1+t^2}\, dt$$

$$= \int_0^1 \left((1+t^2)^2 + \frac{1}{2} e^{1+t^2} \right) 2t\, dt$$

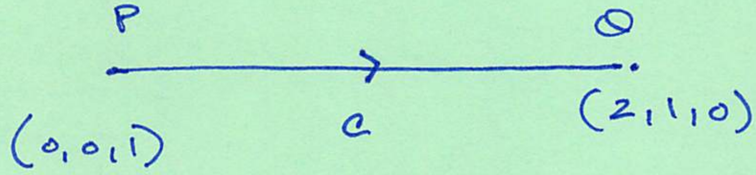
$u = 1+t^2, \quad du = 2t\,dt, \quad u(0) = 1, \quad u(1) = 2$

$$= \int_1^2 u^2 + \frac{1}{2} e^u \, du$$

$$= \frac{1}{3}u^3 + \frac{1}{2}e^u \Big]_1^2 = \frac{8}{3} + \frac{1}{2}e^2 - \left(\frac{1}{3} + \frac{1}{2}e\right)$$

$$= \frac{7}{3} + \frac{1}{2}\left(e^2 - e\right)$$

#41. $\vec{F} = \langle x - y^2,\ y - z^2,\ z - x^2 \rangle$



$$C: \quad \vec{r}(t) = t\langle 2,1,0\rangle + (1-t)\langle 0,0,1\rangle, \quad 0 \le t \le 1$$

$$= \langle 2t, t, 0\rangle + \langle 0, 0, 1-t\rangle$$

$$= \langle 2t, t, 1-t\rangle$$

$$\vec{r}\,'(t) = \langle 2, 1, -1\rangle$$

$$\vec{F}(\vec{r}(t)) = \left\langle 2t - (t)^2,\ t - (1-t)^2,\ (1-t) - (2t)^2 \right\rangle$$

$$= \left\langle 2t - t^2,\ t - 1 + 2t - t^2,\ 1 - t - 4t^2 \right\rangle$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = 2(2t - t^2) + (-t^2 + 3t - 1) + (-4t^2 - t + 1)$$

$$= -7t^2 + 6t$$

$$\text{work} = \int_0^1 -7t^2 - 6t = \frac{2}{3}$$