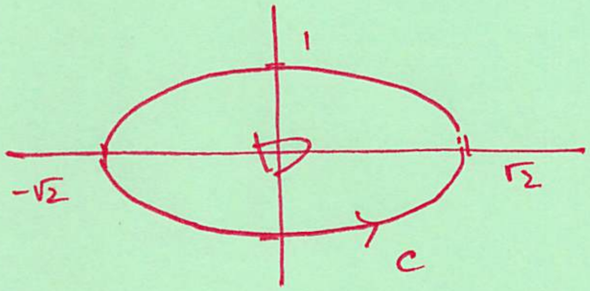
\# 16.4.8 $\int_C y^4 \, dx + 2xy^3 \, dy$



$$= \iint_D \frac{\partial}{\partial x}\left[2xy^3\right] - \frac{\partial}{\partial y}\left[y^4\right] dA$$

$$= \iint_D 2y^3 - 4y^3 \, dA$$

$$= -2 \iint_D y^3 \, dA$$

$$\frac{x^2}{2} + y^2 = 1$$

$$\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{1}\right)^2 = 1$$

Let $u = \frac{x}{\sqrt{2}}$, $v = y$

$$\Longrightarrow \quad u^2 + v^2 = 1$$

$\Longrightarrow \quad x = \sqrt{2}\,u \quad \text{and} \quad y = v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{vmatrix} = \sqrt{2}$$

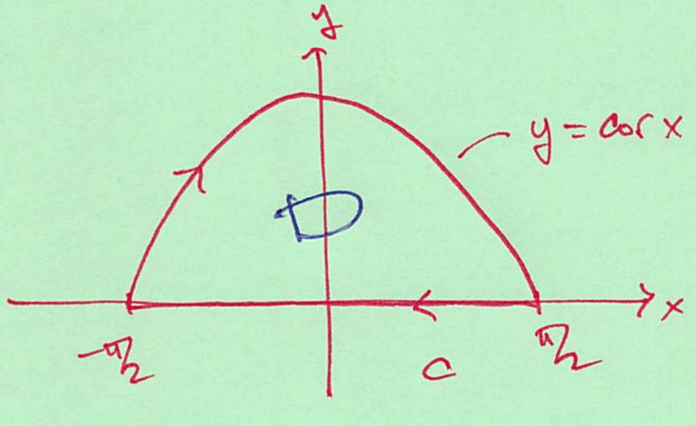
$$\text{Integral} = -2 \iint_{u^2+v^2 \leq 1} v^3 \cdot \sqrt{2}\, du\, dv$$

$$= -2\sqrt{2} \int_0^{2\pi} \int_0^1 r^3 \sin^3\phi \cdot r\, dr\, d\theta$$

full period

$$= 0$$

16.4.12 $\vec{F} = \langle e^{-x} + y^2,\ e^{-y} + x^2 \rangle$



$$\int_{-C} \vec{F} \cdot d\vec{r} = \iint_D \operatorname{curl} \vec{F}\, dA$$

$$= \iint_D \frac{\partial}{\partial x}\left[e^{-y} + x^2\right] - \frac{\partial}{\partial y}\left[e^{-x} + y^2\right] dA$$

$$= \iint_D 2x - 2y \, dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} 2x - 2y \, dy\, dx$$

$$= 2\int_{-\pi/2}^{\pi/2} xy - \frac{y^2}{2}\Big|_0^{\cos x}\,dx$$

$$= \int_{-\pi/2}^{\pi/2} \underbrace{2x\cos x}_{\text{odd}} - \underbrace{\cos^2 x}_{\text{even}}\,dx$$

$$= -\int_0^{\pi/2} 1 + \cos 2x\,dx$$

$$= -\left[x + \cancel{\tfrac{1}{2}\sin 2x}\right]_0^{\pi/2}$$

$$= -\frac{\pi}{2}$$

$$\therefore \int_C \vec{F}\cdot d\vec{r} = -\left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$$

16.4.27

$$F(x,y) = \frac{2xy\,\hat{\imath} + (y^2-x^2)\,\hat{\jmath}}{(x^2+y^2)^2}$$

Q

$$\frac{d}{dx}\left[\frac{y^2-x^2}{(x^2+y^2)^2}\right] = \frac{(x^2+y^2)^2(-2x) - (y^2-x^2)\left[2(x^2+y^2)^1 \cdot 2x\right]}{(x^2+y^2)^4}$$

$$= \frac{(x^2+y^2)(-2x) - (y^2-x^2)(4x)}{(x^2+y^2)^3}$$

$$= \frac{-2x^3 - 2xy^2 - 4xy^2 + 4x^3}{(x^2+y^2)^3}$$

$$= \frac{2x^3 - 6xy^2}{(x^2+y^2)^3}$$

P

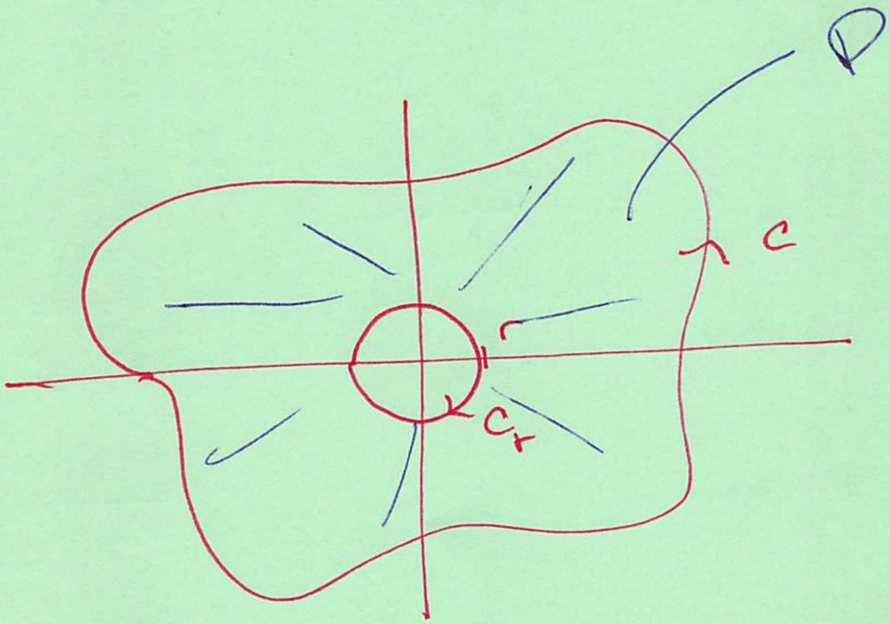
$$\frac{d}{dy}\left[\frac{2xy}{(x^2+y^2)^2}\right] = \frac{(x^2+y^2)^2(2x) - (2xy)\left[2(x^2+y^2)^1 \cdot (2y)\right]}{(x^2+y^2)^4}$$

$$= \frac{(x^2+y^2)(2x) - 8xy^2}{(x^2+y^2)^3}$$

$$= \frac{2x^3 + 2xy^2 - 8xy^2}{(x^2+y^2)^3}$$

$$= \frac{2x^3 - 6xy^2}{(x^2+y^2)^3}$$

$$\therefore \text{curl } \vec{F} = Q_x - P_y = 0.$$



$$\int_{\partial D} \vec{F}\cdot d\vec{r} = \iint_{D} \operatorname{curl}\vec{F}\, dA = 0$$

$$\Rightarrow \int_{C} \vec{F}\cdot d\vec{r} + \int_{C_r} \vec{F}\cdot d\vec{r} = 0$$

$$\Rightarrow \int_{C} \vec{F}\cdot d\vec{r} = -\int_{C_r} \vec{F}\cdot d\vec{r} = \int_{-C_r} \vec{F}\cdot d\vec{r}$$

$$-C_r : \quad \vec{r}(t) = \langle r\cos t, r\sin t\rangle$$

$$0 \le t \le 2\pi$$

$$\vec{r}'(t) = \langle -r\sin t, r\cos t\rangle$$

$\longrightarrow$

$$\vec{F}(\vec{r}(t)) = \frac{2(r\cos t)(r\sin t)\hat{i} + \left(r^2\sin^2 t - r^2\cos^2 t\right)\hat{j}}{(r^2)^2}$$

$$= \frac{\sin(2t)\hat{i} + (-\cos(2t))\hat{j}}{r^2}$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)$$

$$= -\frac{1}{r}\sin t \sin(2t) - \frac{1}{r}\cos t \cos 2t$$

$$= -\frac{1}{r}\left(\cos(2t - t)\right)$$

$$= -\frac{1}{r}\cos t$$

$$\therefore \int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \left(-\frac{1}{r}\underbrace{\cos t}_{\text{full period}}\right) dt = 0$$