

#1.1.6

$$\begin{aligned}x + y - 2z &= 5 \\ 2x + 3y + 4z &= 2 \quad 2R_1 - R_2\end{aligned}$$

$$\begin{aligned}\rightarrow \quad x + y - 2z &= 5 \\ -y - 8z &= 8 \quad R_1 + R_2\end{aligned}$$

$$\begin{aligned}\rightarrow \quad x + y - 2z &= 5 \quad 5R_1 - R_2 \\ -10z &= 13\end{aligned}$$

$$\begin{aligned}\rightarrow \quad 5x + 5y &= -3 \\ -10z &= 13\end{aligned}$$

$$\therefore z = -13/10 \text{ and}$$

$$x = -y - \frac{3}{5}$$

y free

1.3.14

$$(a) A(BD)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} (1)(3) + (0)(2) & (1)(-2) + (0)(5) \\ (2)(3) + (1)(2) & (2)(-2) + (1)(5) \\ (3)(3) + (2)(2) & (3)(-2) + (2)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 8 & 1 \\ 13 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(3) + (2)(8) + (3)(13) & (1)(-2) + (2)(1) + (3)(4) \\ (2)(3) + (1)(8) + (4)(13) & (2)(-2) + (1)(1) + (4)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 58 & 12 \\ 66 & 23 \end{bmatrix}$$

$$(b) (AB)D$$

$$= \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \right) \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1) + (2)(2) + (3)(3) & (1)(0) + (2)(1) + (3)(2) \\ (2)(1) + (1)(2) + (4)(3) & (2)(0) + (1)(1) + (4)(2) \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (14)(3) + (8)(2) & (14)(-2) + (8)(5) \\ (16)(3) + (9)(2) & (16)(-2) + (9)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 58 & 12 \\ 66 & 13 \end{bmatrix}$$

$$(2) A(C+E)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(5) + (2)(4) + (3)(5) & (1)(-5) + (2)(2) + (3)(3) & (1)(8) + (2)(9) + (3)(4) \\ (2)(5) + (1)(4) + (4)(5) & (2)(-5) + (1)(2) + (4)(3) & (2)(8) + (1)(5) + (4)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$$

(d)

$$AC = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(3) + (2)(4) + (3)(2) & (1)(-1) + (2)(1) + (3)(1) & (1)(3) + (2)(5) + (3)(3) \\ (2)(3) + (1)(4) + (4)(2) & (2)(-1) + (1)(1) + (4)(1) & (2)(3) + (1)(5) + (4)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 4 & 22 \\ 18 & 3 & 23 \end{bmatrix}$$

$$AE = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2) + (2)(0) + (3)(3) & (1)(-4) + (2)(1) + (3)(2) & (1)(5) + (2)(4) + (3)(1) \\ (2)(2) + (1)(0) + (4)(3) & (2)(-4) + (1)(1) + (4)(2) & (2)(5) + (1)(4) + (4)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 & 16 \\ 16 & 1 & 18 \end{bmatrix}$$

$$\Rightarrow AC + AE = \begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$$

$$(e) (2AB)^T$$

$$= \left(2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \right)^T$$

$$= \left(2 \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix} \right)^T$$

$$= \begin{pmatrix} 28 & 16 \\ 32 & 18 \end{pmatrix}^T$$

$$= \begin{bmatrix} 28 & 32 \\ 16 & 18 \end{bmatrix}$$

$$2(AB)^T = 2 \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}^T$$

$$= 2 \begin{bmatrix} 14 & 16 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 28 & 32 \\ 16 & 18 \end{bmatrix}$$

$$4) A(C-3E)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - 3 \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \left(\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -12 & 15 \\ 0 & 3 & 12 \\ 9 & 6 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} -3 & 11 & -12 \\ 4 & -2 & -9 \\ -7 & -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(-3)(2)(4) + (3)(-7) & (1)(11) + (2)(-2) + (3)(-5) & (1)(-12) + (2)(-7) + (3)(0) \\ (2)(-7) + (1)(4) + (4)(-7) & (2)(11) + (1)(-2) + (4)(-5) & (2)(-12) + (1)(-7) + (4)(0) \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -8 & -26 \\ -30 & 0 & -31 \end{bmatrix}$$

1.4.10

$$(1) \quad A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(2) \quad A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

1.4.12

$$(1) A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(2) A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

#1.4.32

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$$

then $A \neq 0$ and $B \neq C$. And.

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow AB = AC.$$