1.5.36 $A\vec{x} = \vec{b} \iff \vec{x} = A^{-1}b$

(a) $$\vec{x} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 16 \\ 22 \end{bmatrix}$$

(b) $$\vec{x} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 15 \end{bmatrix} = \begin{bmatrix} 38 \\ 53 \end{bmatrix}$$

1.5.38

$$A^2x = b \iff (AA)x = b$$

$$\iff A(Ax) = b$$

$$\iff Ax = A^{-1}b$$

$$\iff x = A^{-1}(A^{-1}b)$$

$$\iff x = (A^{-1}A^{-1})b$$

$$\iff x = (A^{-1})^2 b$$

$$x = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 \\ -6 \end{bmatrix}$$

1.5.40

$$C^T A x = b \iff C^T(Ax) = b$$

$$\iff Ax = (C^T)^{-1} b$$

$$\iff Ax = (C^{-1})^T b$$

$$\iff x = A^{-1}(C^{-1})^T b$$

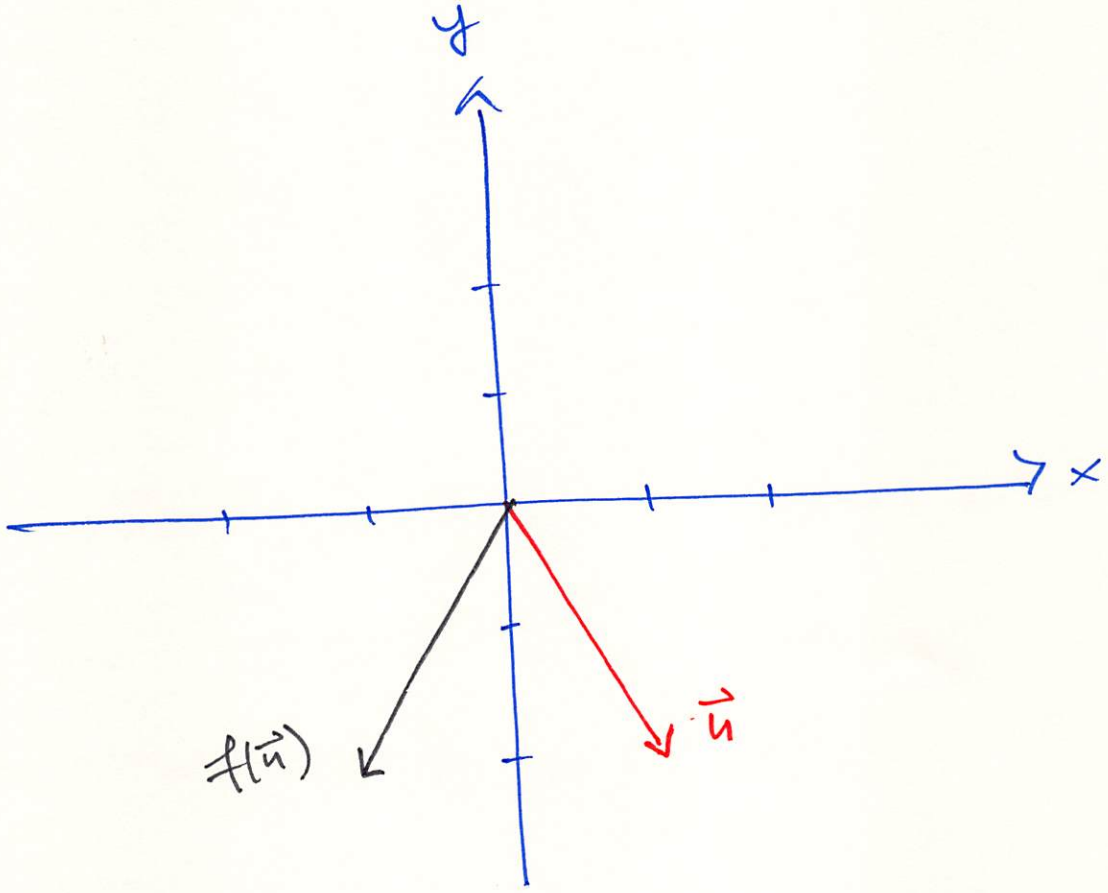
$$x = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 9 \end{bmatrix}$$

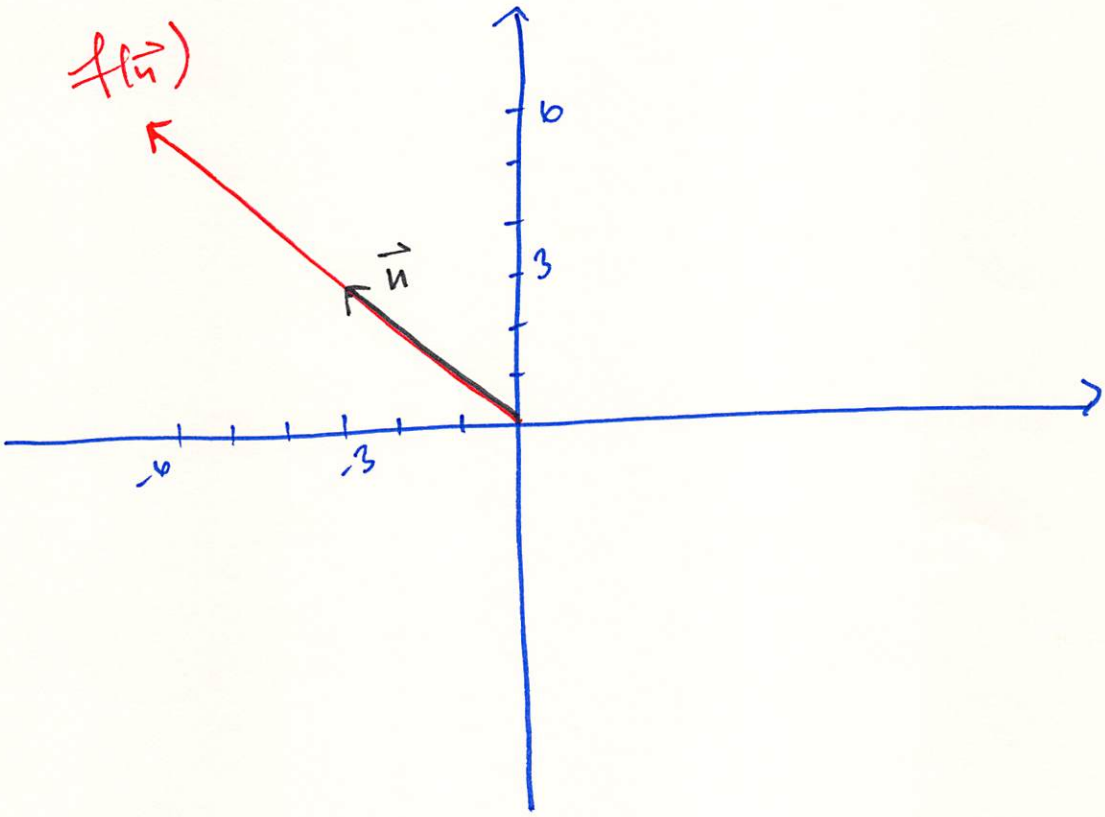
#1.6.2 $f: \mathbb{R}^2 \to \mathbb{R}^2$, $f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$$f(\vec{u}) = f\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$



#16.6 $f: \mathbb{R}^2 \to \mathbb{R}^2$, $f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$$f(\vec{u}) = f\begin{bmatrix} -3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 3 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \end{bmatrix}$$



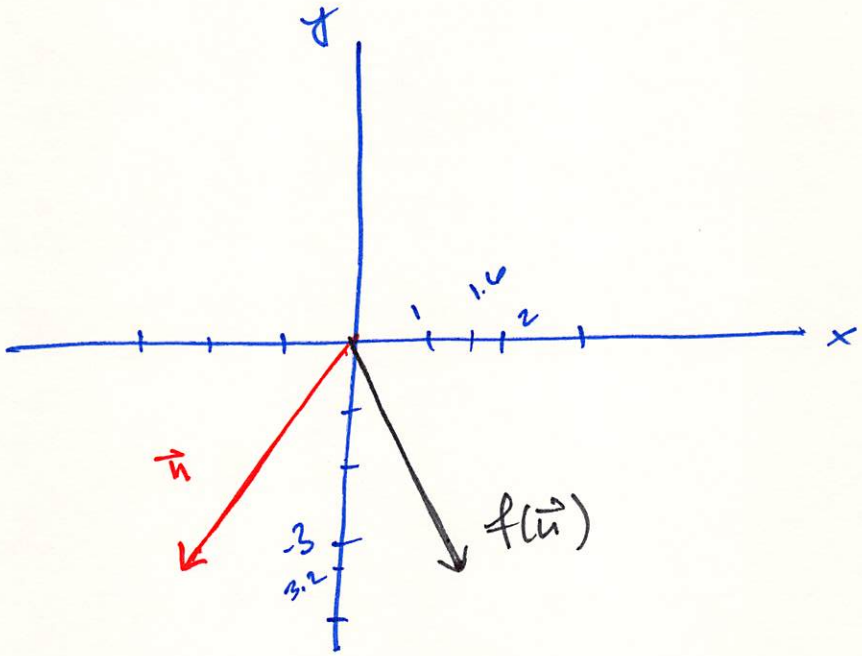
#1.6.4. $f : \mathbb{R}^2 \to \mathbb{R}^2$

$$f(\vec{x}) = \begin{bmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{bmatrix} \vec{x}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \vec{x}$$

$$f(\vec{u}) = f\begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3\sqrt{3}-2}{2} \\ \frac{-2\sqrt{3}-3}{2} \end{bmatrix} \approx \begin{bmatrix} 1.6 \\ -3.2 \end{bmatrix}$$



#1.6.10 $f: \mathbb{R}^2 \to \mathbb{R}^3$, $f(\vec{x}) = A\vec{x}$

where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Let $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

$\vec{w} \in \text{Range}(f)$

$\iff$ there is $\vec{x} \in \mathbb{R}^2$ s.t. $f(\vec{x}) = \vec{w}$

$\iff$ there is $\vec{x} \in \mathbb{R}^2$ s.t. $A\vec{x} = \vec{w}$

$\iff$ the system $A\vec{x} = \vec{w}$ has a solution.

$$\left.\begin{aligned} x + 2y &= 1 \\ y &= 1 \\ x + y &= 1 \end{aligned}\right\} \iff \begin{aligned} &x + 2(1) = 1 \\ &\text{and} \\ &x + (1) = 1 \end{aligned} \iff \begin{aligned} &x = -1 \\ &\text{and} \\ &x = 0 \end{aligned}$$

impossible

$\therefore$ the system doesn't have a solution.