4.6.2

(b) $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

Since $|S| = 4$ and $\dim(\mathbb{R}^3) = 3$, then

S is L.D. Hence, S is not a basis.

(c) $S = \left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

Check L.I:

$$c_1 \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Longleftrightarrow \underbrace{\begin{bmatrix} 3 & -1 & 0 \\ 2 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $\det A = -9$ $\Rightarrow$ A is invertib

$\Rightarrow$ system has only trivial solution

$\Rightarrow$ S is L.I.

Since $|S| = 3$, S is L.I, and $\dim(\mathbb{R}^3) = 3$ then S is a basis.

#4.6.4

(a) $S = \{-t^2+t+2,\ 2t^2+2t+3,\ 4t^2-1\}$

Check L.I. :

$$c_1(-t^2+t+2)+c_2(2t^2+2t+3)+c_3(4t^2-1)=0$$

$$\Leftrightarrow\ (-c_1+2c_2+4c_3)t^2+(c_1+2c_2)t+(2c_1+3c_2-c_3)=0$$

$$\Leftrightarrow\ \underbrace{\begin{bmatrix} -1 & 2 & 4 \\ 1 & 2 & 0 \\ 2 & 3 & -1 \end{bmatrix}}_{A}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $\det(A)=0$ $\Rightarrow$ A is not invertible

$\Rightarrow$ system has nontrivial solutions

$\Rightarrow$ vectors are L.D.

$\Rightarrow$ S is not a basis.

(b) $S = \{ t^2 + 2t - 1, \quad 2t^2 + 3t - 2 \}$

Since $|S| = 2$ and $\dim(P_2) = 3$,

then the vectors cannot span.

Hence, S is not a basis.

#4.7.22 $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 2 \\ 1 & -3 & 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

$$[A \mid \vec{b}] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

z is free, $x = -2z$, $y = -1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2z \\ -1 \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}}_{x_p} + \underbrace{z\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}}_{x_h}$$