$\# 4.8.4 \quad V = P_2, \quad S = \{t^2 - t + 1,\ t+1,\ t^2+1\},$

$v = 4t^2 - 2t + 3$

$$c_1(t^2 - t + 1) + c_2(t+1) + c_3(t^2+1) = 4t^2 - 2t + 3$$

$$(c_1 + c_3)t^2 + (-c_1 + c_2)t + (c_1 + c_2 + c_3) = 4t^2 - 2t + 3$$

$$\begin{aligned} c_1 + c_3 &= 4 \\ -c_1 + c_2 &= -2 \\ c_1 + c_2 + c_3 &= 3 \end{aligned} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ -1 & 1 & 0 & -2 \\ 1 & 1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\therefore [v]_S = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

4.8.10 $V = P_2$, $S = \{t^2+1,\ t+1,\ t^2+t\}$

$$[v]_S = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

$$\Rightarrow v = 3(t^2+1) + (-1)(t+1) + (-2)(t^2+t)$$

$$= 3t^2 + 3 - t - 1 - 2t^2 - 2t$$

$$= t^2 - 3t + 2$$

4.8.32. Find an isomorphism $L: P_2 \to \mathbb{R}^3$.

Let $S = \{t^2, t, 1\}$ be the standard basis of P_2. Let $p(t) = at^2 + bt + c$ be any polynomial in P_2, then the coordinate map

$$L(p(t)) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

is an isomorphism. (we proved this in lecture).

#4.9. 6 (a)

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 5 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

$$\Rightarrow \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & -33/7 \\ 0 & 1 & 0 & 23/7 \\ 0 & 0 & 1 & -8/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \left\{ \begin{bmatrix} 1 & 0 & 0 & -33/7 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 23/7 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & -8/7 \end{bmatrix} \right\}$$

is a basis for the row space of A.

4.9.8 (b)

$$A = \begin{bmatrix} -2 & 2 & 3 & 7 & 1 \\ -2 & 2 & 4 & 8 & 0 \\ -3 & 3 & 2 & 8 & 4 \\ 4 & -2 & 1 & -5 & -7 \end{bmatrix}$$

$$\Longrightarrow \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & +1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Longrightarrow \left\{ \begin{bmatrix} -2 \\ -2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \end{bmatrix} \right\} \text{ is a}$$

basis for the column space of A.