#5.1.2 (b)

$$\left\| \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix} \right\| = \sqrt{(-1)^2 + (-3)^2 + (-4)^2}$$

$$= \sqrt{1 + 9 + 16}$$

$$= \sqrt{26}$$

#5.1.6 (b) $u = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$$\text{dist}(u, v) = \| u - v \|$$

$$= \left\| \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \right\|$$

$$= \sqrt{(-1)^2 + (-1)^2 + (-1)^2}$$

$$= \sqrt{3}$$

5.1.6 (b)

$$\langle u, v \rangle = u \cdot v = 0 + 2 + 0 = 2$$

$$\|u\| = \sqrt{0+1+1} = \sqrt{2}$$

$$\|v\| = \sqrt{1+4+0} = \sqrt{5}$$

$$\cos\theta = \frac{\langle u, v \rangle}{\|u\| \cdot \|v\|} = \frac{2}{\sqrt{2}\sqrt{5}} = \frac{2}{\sqrt{10}}$$

#5.3.10 (a) $f(t) = 1+t$, $g(t) = 2-t$

$$\langle f, g \rangle = \int_0^1 (1+t)(2-t)\,dt$$

$$= \int_0^1 2 + t - t^2 \,dt$$

$$= \left[2t + t^2/2 - t^3/3 \right]_0^1$$

$$= 2 + 1/2 - 1/3$$

$$= \frac{6 + 3 - 2}{6}$$

$$= 7/6$$

5.3.24

$$\left\langle \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right\rangle = u_1 v_1 - u_2 v_1 - u_1 v_2 + 3 u_2 v_2$$

Let $S = \{\hat{i}, \hat{j}\}$.

$$c_{11} = \langle \hat{i}, \hat{i} \rangle = 1.1 - 0 - 0 + 3.0 = 1$$

$$c_{12} = \langle \hat{i}, \hat{j} \rangle = 0 - 0 - 0 + 0 = 0 = c_{21}$$

$$c_{22} = \langle \hat{j}, \hat{j} \rangle = 0 - 0 - 0 + 3 = 3$$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$V = \mathbb{R}_2 , \quad u = [u_1 \ u_2], \quad v = [v_1 \ v_2]$

$\langle u, v \rangle = u_1 v_1 - u_2 v_1 - u_1 v_2 + 5 u_2 v_2$

Let $S = \{ \hat{i}^T, \hat{j}^T \} = \{ [1 \ 0], [0 \ 1] \}$

$c_{11} = \langle \hat{i}^T, \hat{i}^T \rangle = 1 - 0 - 0 + 0 = 1$

$c_{12} = \langle \hat{i}^T, \hat{j}^T \rangle = 0 - 0 - 0 + 0 = 0 = c_{21}$

$c_{22} = \langle \hat{j}^T, \hat{j}^T \rangle = 0 - 0 - 0 + 5$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$