#5.4.8 $\quad S = \{\overset{u_1}{t}, \overset{u_2}{e^t}\}$

$T' = \{v_1, v_2\}$ orthogonal.

Set $v_1 = u_1 = t$

Set $v_2 = u_2 - \operatorname{proj}_{v_1}(u_2)$

$$= u_2 - \frac{\langle u_2, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1$$

$$\langle u_2, v_1 \rangle = \int_0^1 e^t \cdot t \, dt = te^t - e^t \Big]_0^1$$

$$= (e - e) - (0 - e^0) = 1$$

$$\langle v_1, v_1 \rangle = \int_0^1 t^2 \, dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\Rightarrow \quad u_2 = e^t - \left(\frac{1}{1/3}\right) t = e^t - 3t$$

$$\langle e^t, e^t \rangle = \int_0^1 e^{2t}\, dt = \frac{1}{2} e^{2t} \Big|_0^1$$

$$= \frac{1}{2} e^2 - \frac{1}{2}$$

$$\langle u_2, u_2 \rangle = \langle e^t - 3t,\ e^t - 3t \rangle$$

$$= \langle e^t, e^t \rangle - 2 \langle e^t, -3t \rangle + \langle -3t, -3t \rangle$$

$$= \langle e^t, e^t \rangle + 6 \langle e^t, t \rangle + 9 \langle t, t \rangle$$

$$= \frac{1}{2} e^2 - \frac{1}{2} + 6 \cdot 1 + 9 \cdot \frac{1}{3}$$

$$= \frac{1}{2} e^2 + 9 - \frac{1}{2}$$

$$= \frac{1}{2} e^2 + \frac{15}{2} = \frac{e^2 + 15}{2}$$

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{t}{\sqrt{1/3}} = \sqrt{3}\, t$$

$$w_2 = \frac{v_2}{\|v_2\|} = \frac{e^t - 3t}{\sqrt{\frac{e^2+15}{2}}}$$

#5.4.10 $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

$\quad\quad u_1 \quad\quad u_2 \quad\quad u_3$

$v_1 = u_1$

$$v_2 = u_2 - \text{proj}_{v_1}(u_2) = u_2 - \frac{\langle u_2, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \left(\frac{2}{3}\right) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$3v_2 = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

reset $v_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$$v_3 = u_3 - \text{proj}_{v_1}(u_3) - \text{proj}_{v_2}(u_3)$$

$$v_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \left(\frac{6}{3}\right)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \left(\frac{3}{6}\right)\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$2v_3 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} - \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \leftarrow \text{reset } v_3$$

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$w_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$w_3 = \frac{v_3}{\|v_3\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

#5.4.21 (a)

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$S = \left\{ \underset{u_1}{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}, \underset{u_2}{\begin{bmatrix} 2 \\ 3 \end{bmatrix}} \right\}$$

$$v_1 = u_1$$

$$v_2 = u_2 - \text{proj}_{v_1}(u_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \left(\frac{-1}{2}\right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$2v_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \leftarrow \text{reset } v_2$$

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$w_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{89}} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$T = \{w_1, w_2\}$$

$$\langle w_1, u_1 \rangle = \frac{1}{\sqrt{2}}(1+1) = \frac{2}{\sqrt{2}}$$

$$\langle w_1, u_2 \rangle = \frac{1}{\sqrt{2}}(2-3) = -\frac{1}{\sqrt{2}}$$

$$\langle w_2, u_2 \rangle = \frac{1}{\sqrt{89}}(10+24) = \frac{34}{\sqrt{89}}$$

$$Q = [w_1, w_2] = \begin{bmatrix} 1/\sqrt{2} & 5/\sqrt{89} \\ -1/\sqrt{2} & 8/\sqrt{89} \end{bmatrix}$$

$$R = \left[[u_1]_T \; [u_2]_T \right] = \begin{bmatrix} 2/\sqrt{2} & -1/\sqrt{2} \\ 0 & 34/\sqrt{89} \end{bmatrix}$$

$$A = QR.$$