Math 307 Exam 1 Review, Spring 2024

Name:

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
Total:	0	

- You have 75 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work, unless otherwise indicated. You will get almost no credit for solutions that are not fully justified.
- No electronic devices are authorized.
- You may use a 3x5 notecard with notes, no other resources are authorized.
- The back side of each page can be used as scratch work or for longer solutions. Please indicate clearly if your solution extends to the back side of a page.

1. True/False questions. No justification necessary.

(a) True False There is a 5×6 matrix that row-equivalent to a matrix with 6 leading 1's.

(b) True False If $f(\vec{x}) = A\vec{x}$ is a 2×2 matrix transformation, then the image of every 2-vector is a linear combination of $f(\mathbf{i})$ and $f(\mathbf{j})$.

(c) True False If A is a 3×2 matrix and B is a 3×2 matrix, then AB is a 3×2 matrix.

(d) True False If A is a square matrix, then $(2A)^4 = 16A^4$.

(e) True False If A is an invertible matrix, then the homogeneous system $A\vec{x} = \vec{0}$ only has the trivial solution.

(f) True False The matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

is invertible.

(g) True False The transpose of a 7×18 matrix is a matrix of size 18×7.

(h) True False For all $n \times n$ invertible matrices A and B, $(AB)^{-1} = B^{-1}A^{-1}$.

(i) True False If every entry of a 17×17 matrix is a 2, then the determinant of this matrix is 2^{17}.

(j) True False $\det(AA^T) = \det(A^2)$.

(k) True False The determinant of a triangular matrix is the product of the diagonal entries.

(l) True False For all square matrices A and B, $\det(A + B) = \det(A) + \det(B)$.

(m) True False If A row reduces to B, then B row reduces to A.

(n) True False To add 2 times row 3 onto row 1 of the 3×3 matrix A we multiply on the right by the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(o) True False Every matrix in row-reduced echelon form has at least one 1 in one of its columns.

(p) True False The only 1×1 matrices that are in row-reduced echelon form are $[0]$ and $[1]$.

(q) True False A matrix of size 3×4 can have 4 leading 1's.

(r) True False If $\det(A) = 0$, then A is invertible.

(s) True False There exist nonzero square matrices A and B such that $(A+B)^2 = A^2 + B^2$.

(t) True False Multiplying a 3×3 matrix A on the left by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

has the effect of scaling every entry in row 2 of A by 7.

(u) True False There is a 2×2 invertible matrix that has 3 entries that are 0.

(v) True False There is a 3×4 matrix A and a 4×3 matrix B such that $AB = BA$.

(w) True False A homogeneous system of linear equations $A\vec{x} = \vec{0}$ is always consistent.

(x) True False If A is not invertible, then A can be row reduced to a matrix with a row of zeros.

(y) True False For any matrix A, the matrix AA^T is symmetric.

(z) True False For each $n \times n$ matrix A we have $\det(2A) = 2\det(A)$.

2. Which of the following matrices are in RREF. No justification is necessary.

Matrix	Is in RREF	Is NOT in RREF
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		

3. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Compute the following or state that the expression is not defined.

(a) AC

(b) BC

(c) $A+C$

(d) $A+C^T$.

(e) $(BA)^T+2C$

4. Solve the linear system of equations

$$\begin{aligned} x_1 - 7x_2 + x_5 &= 3 \\ x_3 - x_5 &= 2 \\ x_4 + x_5 &= 1 \end{aligned}$$

5. The coefficient matrix of each of the following augmented matrices is in row-reduced echelon form. In the space provided write the solution set (if it exists) to the corresponding system of linear equations

Matrix	Solution Set
$\left[\begin{array}{ccccc\|c} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$	
$\left[\begin{array}{ccccc\|c} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$	
$\left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$	

6. Let

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 4 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

(a) Show that A is invertible three different ways.

(b) Solve the linear system $(3A^2)^T\vec{x} = \vec{b}$.

7. Compute the determinant of the following matrix by reducing to triangular form.

$$C = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 8 & 4 & 7 & 0 \\ 1 & 1 & 0 & -1 \\ 2 & 3 & 0 & 0 \end{bmatrix}$$

8. Let

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 4 & 11 \\ 0 & 0 & 3 & 17 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 0 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{bmatrix}$$

Compute the following determinants:

(a) $\det A$, $\det B$, and $\det C$.

(b) $\det(ABABAB)$, $\det(A^T B^{-1} C^2)$

9. Let $f : R^3 \to \mathbb{R}^3$ be the matrix transformation $f(\vec{x}) = A\vec{x}$ where

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ -1 & 1 & -3 \end{bmatrix}.$$

(a) Sketch the image of the vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

(b) Is the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$ in the range of f?

(c) The volume of a parallelepiped S is 5, what is the volume of its image $f(S)$?

10. For each of the following, state that the property holds, or give an example of why the property fails. You may assume that the matrices are all of the appropriate size.

(a) $(A + B)(D + E) = AD + AE + BD + BE$

(b) $(CA = BA) \Rightarrow C = B$

(c) $AB = BA$

(d) $AB = O \Rightarrow (A = O \text{ or } B = O)$