Name: Solutions Section: 7 8

Evaluate the following integral using a trigonometric substitution.

$$\int \sqrt{25 - x^2}\, dx$$

You will need the following three trig identities for this problem.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$
$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Pattern $a^2 - x^2$

Use $x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

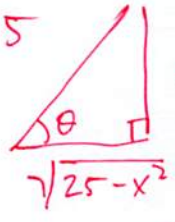
$\sqrt{25 - x^2} = \sqrt{25 - 25\sin^2\theta}$

$= 5\sqrt{1 - \sin^2\theta}$

$= 5\sqrt{\cos^2\theta}$

$= 5\cos\theta$

$x = 5\sin\theta \Rightarrow \sin\theta = \frac{x}{5} = \frac{opp}{hyp}$



& Pyth.

$\Rightarrow \cos\theta = \frac{\sqrt{25 - x^2}}{5}$

$$\int \sqrt{25 - x^2}\, dx$$

$$= \int 5\cos\theta \cdot 5\cos\theta\, d\theta$$

$$= 25 \int \cos^2\theta\, d\theta$$

$$= 25 \int \frac{1 + \cos(2\theta)}{2}\, d\theta$$

$$= \frac{25}{2}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$$

$$= \frac{25}{2}\left[\theta + \frac{2\sin\theta\cos\theta}{2}\right] + C$$

$$= \frac{25}{2}\left[\sin^{-1}\left(\frac{x}{5}\right) + \frac{x}{5}\cdot\frac{\sqrt{25 - x^2}}{5}\right] + C$$