Name: Solutions Section: 7 8

1. Find $\int_2^\infty \frac{1}{1-x}\,dx$.

$$= \lim_{t\to\infty}\Big[-\ln|1-x|\Big]_2^t$$

$$= \lim_{t\to\infty}\Big[-\ln|1-t| + \ln|1-2|\Big]$$

$$= -\infty.$$

2. How large should $n$ be to guarantee that Simpson's Rule $S_n$ is within $10^{-4}$ of $\int_{-1}^3 xe^x\,dx$. The fourth derivative of $f(x) = xe^x$ is $f^{(4)}(x) = (x+4)e^x$. The error estimate in using Simpson's Rule is

$$|E_S| \le \frac{K(b-a)^5}{180n^4} \text{ where } |f^{(4)}(x)| \le K \text{ for all } a \le x \le b.$$

$$|f^{(4)}(x)| = |(x+4)e^x| = |x+4|\cdot|e^x| \le 7\cdot e^3$$

we may use $K = 7\cdot e^3$

want: $\dfrac{K(b-a)^5}{180n^4} < 10^{-4}$

$$\frac{7\cdot e^3(4)^5}{180n^4} < 10^{-4}$$

solve for $n$

$$\Rightarrow \frac{7\cdot e^3(4)^5}{180}\cdot 10^4 < n^4$$

$$\Rightarrow \sqrt[4]{\frac{7\cdot e^3(4)^5}{180}\cdot 10^4} < n$$

any even integer $n$ satisfying this inequality.