Name: Solutions Section: 7 8

1. Determine if the following geometric series converges or diverges. If it converges find its sum.

$$\sum_{n=2}^{\infty} \frac{3^{n-1}}{4^n}$$

$$\frac{3^{n-1}}{4^n} = \frac{3^n \cdot 3^{-1}}{4^n} = \frac{1}{3} \cdot \left(\frac{3}{4}\right)^n$$

ratio $\left|\frac{3}{4}\right| < 1$

series converges

$$\text{Sum} = \frac{\text{first term}}{1 - \text{ratio}} = \frac{\frac{1}{3} \cdot (3/4)^2}{1 - 3/4}$$

2. Determine if the following telescoping series converges or diverges. If it converges find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{2}{n+1}\right)$$

$$S_n = \left(\frac{2}{1} - \frac{2}{2}\right) + \left(\frac{2}{2} - \frac{2}{3}\right) + \left(\frac{2}{3} - \frac{2}{4}\right) + \cdots + \left(\frac{2}{n} - \frac{2}{n+1}\right)$$

$$S_n = 2 - \frac{2}{n+1}$$

$$\text{Sum} = \lim_{n \to \infty} \left(2 - \frac{2}{n+1}\right) = 2. \quad \text{series converges}$$