Name: Solutions Section: 7 8

Determine if the following series converge absolutely, converge conditionally, or diverge. Clearly state which tests you are using, give full reasoning.

1. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ factorial, try Ratio test

$$\left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right| = \frac{2}{n+1} \to 0 < 1$$

Series converges (absolutely)

First test $\sum |a_n|$.

2. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}-1}$ $\quad \sum_{n=2}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n}-1} \right| = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ DCT with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n}}$ and $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges (p-series $p = 1/2 < 1$)

So $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges.

Next, test $\sum a_n$. Series is alternating with $b_n = \frac{1}{\sqrt{n}-1}$

b_n is pos, dec and $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}-1} = 0$

Therefore, the series converges conditionally