Name: Section: 7 8

Determine the x for which the power series converges absolutely, converges conditionally, diverges. State the interval of convergence and the radius of convergence.

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n \sqrt[3]{n}}$$

Root test:

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(x+2)^n}{3^n \sqrt[3]{n}}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{|x+2|^n}{3^n \sqrt[3]{n}}}$$

$$= \lim_{n\to\infty} \frac{|x+2|}{3 \sqrt[3]{\sqrt[n]{n}}}$$

$$= \frac{|x+2|}{3} \cdot \lim_{n\to\infty} \frac{1}{\sqrt[3]{\sqrt[n]{n}}} = \frac{|x+2|}{3} \cdot \frac{1}{\sqrt[3]{1}}$$

$$= \frac{|x+2|}{3}$$

Solve $L < 1$ radius

$$\frac{|x+2|}{3} < 1 \iff |x+2| < 3 \iff -3 < x+2 < 3$$

$$\iff -5 < x < 1$$ conv. abs. for these x.

Endpt. $x = 1$: $\sum_{n=1}^{\infty} \frac{(1+2)^n}{3^n \sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{3^n}{3^n \cdot \sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

Series diverges (p-series $p = 1/3 < 1$)

Endpt. $x = -5$: $\sum_{n=1}^{\infty} \frac{(-5+2)^n}{3^n \sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{(-3)^n}{3^n \sqrt[3]{n}} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\sqrt[3]{n}}$

the absolute value series diverges by above. Next, try AST with $b_n = \frac{1}{\sqrt[3]{n}}$ pos, dec, $b_n \to 0$ so series converges conditionally.

Answer on Back $\rightarrow$

Abs. Conv. : $-5 < x < 1$

Cond. Conv : $x = -5$

Interval of Conv: $-5 \leq x < 1$

Diverges : elsewhere

$R = 3$