

Name:

Section: 7 8

1. Find the following values.

$$(a) \log_3\left(\frac{1}{27}\right) = x \iff 3^x = \frac{1}{27} \iff x = -3$$

$$(b) \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = x \iff \cos x = -\frac{\sqrt{3}}{2} \iff x = \frac{5\pi}{6}$$

$$(c) \tan^{-1}(\sqrt{3}) = x \iff \tan x = \sqrt{3} \iff x = \frac{\pi}{3}$$

2. Differentiate the following functions.

$$(a) f(x) = \ln(\sin^{-1}(x)) + \sec^{-1}(\sqrt{x}).$$

$$f'(x) = \frac{1}{\sin^{-1}(x)} \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x} \sqrt{(\sqrt{x})^2 - 1}} \cdot \left(\frac{1}{2} x^{-1/2}\right)$$

$$(b) y = \arctan(x^3).$$

$$y' = \frac{1}{1 + (x^3)^2} (3x^2)$$

3. Suppose you have a 200mg sample of cesium-137. The half-life of cesium-137 is 30 years. Find the mass of the sample after 7 years.

$$m(0) = 200$$

$$m(30) = 100 = 200 e^{k \cdot 30}$$

$$\implies \frac{1}{2} = e^{k \cdot 30}$$

$$k \cdot 30 = -\ln 2$$

$$k = -\frac{\ln 2}{30}$$

$$\therefore m = 200 e^{-\ln 2 / 30 \cdot t}$$

$$\implies m(7) = 200 e^{-\ln 2 / 30 \cdot 7} \text{ mg.}$$

4. Evaluate the following integrals.

(a) Find $\int \frac{2x}{1+x^4} dx$. *Hint: let $u = x^2$.*

$$u = x^2$$

$$du = 2x dx$$

$$\rightarrow = \int \frac{du}{1+u^2}$$

$$= \tan^{-1} u + C$$

$$= \tan^{-1}(x^2) + C$$

(b) Evaluate $\int \frac{1}{\sqrt{9-4x^2}} dx$. *Hint: factor 3 out of the denominator and let $u = \frac{2}{3}x$.*

$$\rightarrow = \int \frac{1}{\sqrt{9\left(1 - \frac{4x^2}{9}\right)}} dx$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{1 - \left(\frac{2x}{3}\right)^2}} dx$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{1-u^2}} \left(\frac{3}{2} du\right)$$

$$= \frac{1}{2} \sin^{-1}(u) + C$$

$$= \frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C$$

$$u = \frac{2x}{3}$$

$$du = \frac{2}{3} dx$$

$$\Rightarrow \frac{3}{2} du = dx$$