Name: Solutions Section: 7 8

1. Use integration by parts to evaluate the following integrals.

(a) $\int xe^{3x}dx$

$u = x \qquad dv = e^{3x}dx$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

$$= \frac{xe^{3x}}{3} - \int \frac{1}{3}e^{3x}dx$$

$$= \frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + C$$

(b) $\int x^2 \ln(x)dx$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2 dx$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

2. Use the methods we discussed in lecture to find the following limits.

(a) $\lim\limits_{x\to 0} \dfrac{\sin x}{1-\cos x}$ Type $\frac{0}{0}$

$$\overset{H}{=} \lim_{x\to 0} \frac{\cos x}{1+\sin x} = \frac{\cos(0)}{1+\sin(0)} = \frac{1}{1+0} = 1.$$

(b) $\lim\limits_{x\to 0^+} x\ln(\sin x)$ Type $0\cdot(-\infty)$ use $ab = \frac{b}{1/a}$

$$= \lim_{x\to 0^+} \frac{\ln(\sin x)}{1/x} \quad \text{Type } \frac{-\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{-x^{-2}} = -\lim_{x\to 0^+} \frac{x^2\cos x}{\sin x} \quad \text{Type } \frac{0}{0}$$

$$\overset{H}{=} -\lim_{x\to 0^+} \frac{2x\cos x + x^2(-\sin x)}{\cos x} = \frac{0+0}{1} = 0.$$

(c) $\lim\limits_{x\to\infty} \left(1-\dfrac{1}{x}\right)^{2x}$ Type 1^∞ use ln.

set $L = \lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)^{2x}$

$$\Rightarrow \ln L = \lim_{x\to\infty} 2x\cdot\ln\left(1-\frac{1}{x}\right) \quad \text{Type } 0\cdot\infty \qquad \therefore L = e^{-2}$$

$$= 2\lim_{x\to\infty} \frac{\ln\left(1-\frac{1}{x}\right)}{1/x} \quad \text{Type } \frac{\infty}{\infty}$$

$$\overset{H}{=} 2\cdot\lim_{x\to\infty} \frac{\frac{1}{1-1/x}\cdot(+x^2)}{-x^2} = -2\cdot\left(\frac{1}{1-0}\right) = -2$$