Name: Section: 7 8

1. Evaluate the trigonometric integral $\int \sec^5 x \tan^5 x \, dx$

Odd power tan

use $u = \sec x$

$du = \sec x \tan x \, dx$

$\tan^2 x = \sec^2 x - 1$

$= u^2 - 1$

$= \int \sec^4 x \tan^4 x \cdot \sec x \cdot \tan x \, dx$

$= \int \sec^4 x \cdot (\tan^2 x)^2 \cdot \sec x \tan x \, dx$

$= \int u^4 \cdot (u^2 - 1)^2 \cdot du$

$= \int u^8 - 2u^6 + u^4 \, du$

$= \frac{1}{9}u^9 - \frac{2}{7}u^7 + \frac{1}{5}u^5 + C$

$= \frac{1}{9}\sec^9 x - \frac{2}{7}\sec^7 x + \frac{1}{5}\sec^5 x + C$

2. Evaluate the following integral using a trigonometric substitution $\int x^3\sqrt{1-x^2} \, dx$

Pattern $a^2 - x^2$ use sin

$x = 1 \cdot \sin\theta$

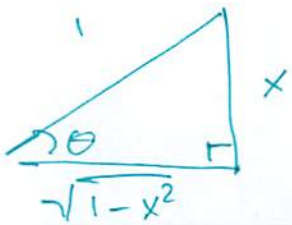
$x^3 = \sin^3\theta$

$dx = \cos\theta \, d\theta$

$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta}$

$= \sqrt{\cos^2\theta}$

$= \cos\theta$



$= \int \sin^3\theta \cdot \cos\theta \cdot \cos\theta \, d\theta$

$= \int \sin^3\theta \cos^2\theta \, d\theta$

$= \int \sin^2\theta \cdot \cos^2\theta \cdot \sin\theta \, d\theta$

$= \int (1-u^2) \cdot u^2 \cdot (-du)$

$= \int u^4 - u^2 \, du$

$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

$= \frac{1}{5}\cos^5\theta - \frac{1}{3}\cos^3\theta + C$

$= \frac{1}{5}\left(\frac{\sqrt{1-x^2}}{1}\right)^5 - \frac{1}{3}\left(\frac{\sqrt{1-x^2}}{1}\right)^3 + C$

odd power sin

$u = \cos\theta$

$du = -\sin\theta$

$\sin^2\theta = 1 - \cos^2 x$

$= 1 - u^2$

3. Evaluate the following integral using trigonometric substitution $\int \frac{dx}{x^2\sqrt{4+x^2}}$

Pattern a^2+x^2 use tan

$x = 2\tan\theta$

$x^2 = 4\tan^2\theta$

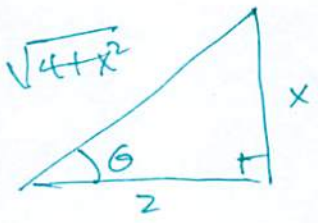
$dx = 2\sec^2\theta\, d\theta$

$\sqrt{4+x^2} = \sqrt{4+4\tan^2\theta}$

$= 2\sqrt{1+\tan^2\theta}$

$= 2\sqrt{\sec^2\theta}$

$= 2\sec\theta$



$\tan\theta = \frac{x}{2} = \frac{opp}{adj}$

$\csc\theta = \frac{hyp}{opp} = \frac{\sqrt{4+x^2}}{x}$

$= \int \frac{2\sec^2\theta\, d\theta}{4\tan^2\theta \cdot 2\sec\theta}$

$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$

$= \frac{1}{4}\int \csc\theta\cot\theta\, d\theta$

$= \frac{1}{4}\left[-\csc\theta\right] + C$

$= -\frac{1}{4}\,\frac{\sqrt{4+x^2}}{x} + C$