

2. Find the following integrals.

(a) Evaluate $\int \frac{x+4}{x^2+5x+6} dx$

$$\frac{x+4}{x^2+5x+6} = \frac{x+4}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\Rightarrow x+4 = A(x+3) + B(x+2)$$

$$x = -3: -3+4 = A \cdot 0 + B(-3+2)$$

$$1 = B(-1)$$

$$B = -1$$

$$x = -2: -2+4 = A(-2+3) + B \cdot 0$$

$$2 = A$$

$$\Rightarrow \int \frac{2}{x+2} + \frac{-1}{x+3} dx$$

$$= 2 \ln|x+2| - 1 \cdot \ln|x+3| + C$$

(b) Evaluate $\int \frac{4x^2+3x+2}{x^3+2x} dx$

$$\frac{4x^2+3x+2}{x^3+2x} = \frac{4x^2+3x+2}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$$\Rightarrow 4x^2+3x+2 = A(x^2+2) + (Bx+C)x$$

$$\begin{aligned} \Rightarrow 4x^2+3x+2 &= Ax^2+2A+Bx^2+Cx \\ &= (A+B)x^2+Cx+2A \end{aligned}$$

$$\Rightarrow A+B=4$$

$$C=3$$

$$2A=2$$

$$\Rightarrow A=1$$

$$\Rightarrow 1+B=4$$

$$\Rightarrow B=3$$

$$\Rightarrow \int \frac{1}{x} + \frac{3x+3}{x^2+2} dx$$

$$= \int \frac{1}{x} dx + 3 \int \frac{x}{x^2+2} dx + 3 \int \frac{1}{x^2+2} dx$$

$$u = x^2+2$$

$$du = 2x dx$$

$$v = \frac{x}{\sqrt{2}}$$

$$dv = \frac{1}{\sqrt{2}} dx$$

$$= \ln|x| + \frac{3}{2} \int \frac{du}{u} + 3\sqrt{2} \int \frac{1}{v^2+1} dv$$

$$= \ln|x| + \frac{3}{2} \ln|u| + 3\sqrt{2} \tan^{-1} v + C$$

$$= \ln|x| + \frac{3}{2} \ln|x^2+2| + 3\sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + C$$

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Section: 7 8

1. Write the abstract partial fraction decomposition for the following rational functions.

$$(a) \frac{x^3 + 9x - 18}{x^2(x-2)(x+2)^3(7x-1)}$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2} + \frac{E}{(x+2)^2} + \frac{F}{(x+2)^3} + \frac{G}{7x-1}$$

$$(b) \frac{x^3 + 9x - 18}{(x^2 - 1)(x^2 + 1)} = \frac{x^3 + 9x - 18}{(x+1)(x-1)(x^2+1)}$$

$$\frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$$

$$(c) \frac{x^3 + 9x - 18}{x^2(x^2 + 1)(x^2 + 2)^3}$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{x^2+2} + \frac{Gx+H}{(x^2+2)^2} + \frac{Ix+J}{(x^2+2)^3}$$