Name: Solutions Section: 7 8

1. Use the Trapezoidal Rule and Simpson's Rule with $n = 4$ to approximate $\int_1^5 x^2 dx$.

$a = 1,\ b = 5,\ n = 4,\ \Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$

x_i	1	2	3	4	5
$y_i = x_i^2$	1	4	9	16	25
T_4 coeff	1	2	2	2	1
$y_i \cdot$ coeff	1	8	18	32	25

mult

$T_4 = \frac{\Delta x}{2}(1 + 8 + 18 + 32 + 25) = \frac{1}{2}(84) = 42$

S_4 coeff	1	4	2	4	1
$y_i \cdot$ coeff	1	16	18	64	25

mult

$S_4 = \frac{\Delta x}{3}(1 + 16 + 18 + 64 + 25) = \frac{1}{3}(124) = 124/3$

2. Find an integer n that would guarantee Simpron's Rule S_n to be within 10^{-8} of $\int_1^5 x^{3/2} dx$. You do not need to simplify your answer.

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2} x^{1/2}$

$f''(x) = \frac{3}{4} x^{-1/2}$

$f'''(x) = -\frac{3}{8} x^{-3/2}$

$f^{(4)}(x) = \frac{9}{16} x^{-5/2}$

$\Rightarrow |f^{(4)}(x)| \le \frac{9}{16} \cdot 1^{-5/2}$ for $1 \le x \le 5$

$\Rightarrow$ we may use $K = 9/16$

we want $\frac{K(b-a)^5}{180 n^4} < 10^{-8}$

solve $\frac{9/16\,(5-1)^5}{180 n^4} < 10^{-8}$

$\Rightarrow \frac{9/16\,(4)^5}{180} \cdot 10^8 < n^4$

$\Rightarrow \sqrt[4]{\frac{9/16\,(4)^5}{180} \cdot 10^8} < n$

3. Evaluate the improper integral $\int_0^\infty \frac{4}{x^2+5x+6}\,dx$.

$$\frac{4}{x^2+5x+6} = \frac{4}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$ clear fractions

$$4 = A(x+3) + B(x+2)$$

clever #'s : $-2, -3$

$x = -2$: $4 = A(-2+3) + B(-2+2) \Rightarrow A = 4$

$x = -3$: $4 = A(-3+3) + B(-3+2) \Rightarrow B = -4$

$$\int \frac{4}{x+2} + \frac{-4}{x+3}\,dx = 4\ln|x+2| - 4\ln|x+3|$$
$$= 4\left(\ln|x+2| - \ln|x+3|\right)$$
$$= 4\ln\left|\frac{x+2}{x+3}\right|$$

$$\Rightarrow \int_0^\infty \frac{4}{x^2+5x+6}\,dx = \lim_{t\to\infty}\int_0^t \frac{4}{x^2+5x+6}\,dx$$
$$= \lim_{t\to\infty}\left[4\ln\left|\frac{x+2}{x+3}\right|\right]_0^t$$
$$= \lim_{t\to\infty}\left(4\ln\left|\frac{t+2}{t+3}\right| - 4\ln(2/3)\right)$$
$$= 4\ln\left|\lim_{t\to\infty}\frac{t+2}{t+3}\right| - 4\ln(2/3)$$
$$\overset{H}{=} 4\cdot\ln\left|\lim_{t\to\infty}\frac{1}{1}\right| - 4\ln(2/3)$$
$$= 4\cdot\ln(1) - 4\ln(2/3) = -4\ln(2/3)$$