Name: Solutions Section: 7 8

1. Evaluate the following improper integrals.

(a) Find $\int_3^4 \frac{1}{\sqrt{x-3}}\, dx$. improper at $x = 3$

$$= \lim_{t \to 3^+} \int_t^4 \frac{1}{\sqrt{x-3}}\, dx$$

$$= \lim_{t \to 3^+} \left[2(x-3)^{1/2} \right]_t^4$$

$$= \lim_{t \to 3^+} \left[2(1)^{1/2} - 2(t-3)^{1/2} \right]$$

$$= 2 - 0 = 2$$

(b) Find $\int_2^\infty \frac{1}{x(\ln x)^3}\, dx$.

Find indefinite integral

$$\int \frac{1}{x(\ln x)^3}\, dx \qquad u = \ln x, \quad du = \frac{1}{x}\, dx$$

$$= \int \frac{1}{u^3}\, du$$

$$= -\frac{1}{2u^2} + C = -\frac{1}{2(\ln x)^2} + C$$

$$= \lim_{t \to \infty} \int_2^t \frac{1}{x(\ln x)^3}\, dx$$

$$= \lim_{t \to \infty} \left[-\frac{1}{2(\ln x)^2} \right]_2^t$$

$$= \lim_{t \to \infty} \left[-\frac{1}{2(\ln t)^2} + \frac{1}{2(\ln 2)^2} \right]$$

$$= 0 + \frac{1}{2(\ln 2)^2} = \frac{1}{2(\ln 2)^2}$$

2. Find the limit of the following sequence.

(a) $\left\{\dfrac{2n+1}{1-4\sqrt{n}}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{2n+1}{1-4\sqrt{n}} \overset{H}{=} \lim_{n\to\infty} \frac{2}{-2n^{-1/2}} = \lim_{n\to\infty}\left(-\sqrt{n}\right) = -\infty$$

Type $\frac{\infty}{-\infty}$

(b) $\left\{\dfrac{n+(-1)^n}{n}\right\}_{n=1}^{\infty}$

Squeeze

$$\frac{n-1}{n} \leq \frac{n+(-1)^n}{n} \leq \frac{n+1}{n}$$

Since $\lim_{n\to\infty} \frac{n-1}{n} = 1 = \lim_{n\to\infty} \frac{n+1}{n}$, then

$$\lim_{n\to\infty} \frac{n+(-1)^n}{n} = 1.$$

(c) $\left\{\left(1+\dfrac{2}{n}\right)^n\right\}_{n=1}^{\infty}$

Common limit $\left(1+\frac{2}{n}\right)^n \to e^2$

(d) $\left\{\left(-\dfrac{1}{3}\right)^n\right\}_{n=1}^{\infty}$

geometric sequence

ratio $= -\frac{1}{3}$ $\Longrightarrow$ $\lim_{n\to\infty}\left(-\frac{1}{3}\right)^n = 0$

$\left|-\frac{1}{3}\right| < 1$

common limit.