Name: Solutions Section: 7 8

1. Find the first five terms of the sequence of n-th partial sums $\{s_n\}$ for the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

$$S_1 = \frac{1}{1} = 1$$

$$S_2 = \frac{1}{1} + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$S_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{3}{2} + \frac{1}{3} = \frac{11}{6}$$

$$S_4 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{11}{6} + \frac{1}{4} = \frac{50}{24} = \frac{25}{12}$$

$$S_5 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{25}{12} + \frac{1}{5}$$

$$= \frac{125 + 12}{60}$$

$$= \frac{137}{60}$$

2. Find the sum of the following series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{8}$

geometric

ratio $= 3 \geq 1$

Series diverges

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 1}{5^n}$ $\displaystyle = \sum_{n=1}^{\infty} \frac{2^n}{5^n} - \sum_{n=1}^{\infty} \frac{1}{5^n}$

sum of geometric series

$$= \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$$

$\dfrac{\text{first term}}{1 - \text{ratio}}$

$$= \frac{(2/5)^1}{1 - 2/5} - \frac{(1/5)^1}{1 - 1/5} = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

(c) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right)$ telescoping

$$S_n = \left(\frac{1}{\ln 2} - \cancel{\frac{1}{\ln 3}}\right) + \left(\cancel{\frac{1}{\ln 3}} - \cancel{\frac{1}{\ln 4}}\right) + \left(\cancel{\frac{1}{\ln 4}} - \cancel{\frac{1}{\ln 5}}\right) + \cdots + \left(\cancel{\frac{1}{\ln n}} - \frac{1}{\ln(n+1)}\right)$$

$$= \frac{1}{\ln 2} - \frac{1}{\ln(n+1)}$$

$$\therefore \sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right) = \lim_{n \to \infty} \left(\frac{1}{\ln 2} - \frac{1}{\ln(n+1)}\right) = \frac{1}{\ln 2}$$

3. Have a nice Spring break!