Name: Solutions Section: 7 8

Determine the convergence for the following series; You may use techniques of geometric series, telescoping series, p-series, divergence test, integral test, comparison tests, absolute convergence test, and alternating series test.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n-1}}$

Alternating Series, try AST

$b_n = \frac{1}{\sqrt{n-1}}$ is pos. & dec.

$b_n = \frac{1}{\sqrt{n-1}} = \frac{1/\sqrt{n}}{\sqrt{1-1/n}} \to \frac{0}{\sqrt{1-0}} = 0$

$\therefore$ series converges

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{7^n+n}$

Direct comparison with $\sum_{n=1}^{\infty} \frac{1}{7^n}$

which converges (geometric $|r = \frac{1}{7}| < 1$).

larger denom. $\hookrightarrow$ $\frac{1}{7^n+n} < \frac{1}{7^n}$

$\therefore$ series converges

3. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1}$

alternating

try AST. $b_n = \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1}$ is pos & dec.

test $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1} = \lim\limits_{n\to\infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1} \cdot \frac{1/\sqrt[3]{n}}{1/\sqrt[3]{n}} = \lim\limits_{n\to\infty} \frac{1}{1-1/\sqrt[3]{n}}$ #4

$= \frac{1}{1-0} = 1$

$\neq 0$

$\therefore \sum_{n=2}^{\infty} (-1)^n \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1}$ diverges by divergence test.

4. $\displaystyle\sum_{n=2}^{\infty} \frac{3\sin n + 4\sin n}{n^2 + \ln n}$ numerator has unpredictable sign

try absolute convergence test.

$$\left| \frac{3\sin n + 4\sin n}{n^2 + \ln n} \right| \leq \frac{7}{n^2}$$

Since $\displaystyle\sum_{n=2}^{\infty} \frac{7}{n^2}$ converges (p-series $p=2$)

then $\displaystyle\sum_{n=2}^{\infty} \left| \frac{3\sin n + 4\sin n}{n^2 + \ln n} \right|$ converges.

Hence, $\displaystyle\sum_{n=2}^{\infty} \frac{3\sin n + 4\sin n}{n^2 + \ln n}$ converges (absolutely).

5. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$

Direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ which diverges

(p-series $p = 1/2 \leq 1$).

$$\frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n} - 1}$$

larger denom.

$\therefore$ series diverges.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n-2}{n^3 - 10}$ Limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

which converges (p-series $p = 2 > 1$).

$$\lim_{n\to\infty} \frac{\left(\frac{n-2}{n^3-10}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{n^3 - 2n^2}{n^3 - 10} = \lim_{n\to\infty} \frac{n^3 - 2n^2}{n^3 - 10} \frac{1/n^3}{1/n^3}$$

$$= \lim_{n\to\infty} \frac{1 - 2/n}{1 - 10/n^3}$$

$$= \frac{1-0}{1-0} = 1 > 0$$

$\therefore$ series converges.