Name: Solutions Section: 7 8

For each of the following power series determine the values of x where the series converges absolutely, converges conditionally, diverges. State the radius of convergence and the interval of convergence. powers of n

1. $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$ Use Root test

$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{2^n x^n}{n^2}\right|} = \lim_{n\to\infty} \frac{2|x|}{(\sqrt[n]{n})^2} = \frac{2|x|}{1} = 2|x|$

Solve $L < 1$ to get interior of I.o.C and R.o.C

$L < 1 \iff 2|x| < 1 \iff |x| < 1/2 \iff -1/2 < x < 1/2$

radius

Endpoint $x = 1/2$: $\sum \frac{2^n \cdot (1/2)^n}{n^2} = \sum \frac{1}{n^2}$ converges absolutely (p-series $p = 2 > 1$)

Endpoint $x = -1/2$: $\sum \frac{2^n(-1/2)^n}{n^2} = \sum \frac{(-1)^n}{n^2}$ converges absolutely by above

Abs conv: $[-1/2, 1/2]$ I.o.C.: $[-1/2, 1/2]$ R.o.C $= 1/2$

cond. conv: nowhere Diverges: elsewhere

2. $\sum_{n=1}^{\infty} \frac{2^n x^n}{n!}$ factorial, Use Ratio test

$L = \lim_{n\to\infty} \left|\frac{2^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n x^n}\right| = \lim_{n\to\infty} \frac{2|x|}{n+1} = 0$ for all x

$L < 1$ for all x.

Abs. conv: all x diverges: nowhere

cond. conv.: nowhere Radius $= \infty$.

I.o.C: $(-\infty, \infty)$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+5)^n}{2^n\sqrt{n}}$ try Ratio test (Root also works)

$$L = \lim_{n\to\infty} \left| \frac{(x+5)^{n+1}}{2^{n+1}\sqrt{n+1}} \cdot \frac{2^n\sqrt{n}}{(x+5)^n} \right| = \lim_{n\to\infty} \frac{|x+5|^{n+1}}{2^{n+1}\cdot\sqrt{n+1}} \cdot \frac{2^n\sqrt{n}}{|x+5|^n}$$

$$= \lim_{n\to\infty} \frac{|x+5|}{2} \cdot \sqrt{\frac{n}{n+1}} = \frac{|x+5|}{2} \cdot \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = \frac{|x+5|}{2}$$

Solve $L < 1$.

$$\frac{|x+5|}{2} < 1 \iff |x+5| < \underbrace{2}_{\text{radius}} \iff -2 < x+5 < 2$$
$$\iff -7 < x < -3$$

<u>Endpoint $x = -3$:</u> $\displaystyle\sum \frac{(-3+5)^n}{2^n\sqrt{n}} = \sum \frac{2^n}{2^n\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$ diverges (p-series $p = \frac{1}{2} < 1$)

<u>Endpoint $x = -7$:</u> $\displaystyle\sum \frac{(-7+5)^n}{2^n\sqrt{n}} = \sum \frac{(-2)^n}{2^n\sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$

Alternating series with $b_n = 1/\sqrt{n}$

b_n's pos, dec, and $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$.

So $\sum \frac{(-1)^n}{\sqrt{n}}$ converges, since $\sum \frac{1}{\sqrt{n}}$ diverges, then the convergence is conditional.

Abs. conv.: $(-7,-3)$

Cond. conv: $x = -7$

I.O.C: $[-7,-3)$

diverges: elsewhere

R.O.C: $R = 2$