Name: Section: 7 8

1. Using the Maclaurin series

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!},$$

   (a) Find a Maclaurin series for $x^2 \sin(x)$.

   (b) Find the sum of the series

$$\frac{\pi}{4} - \frac{\pi^3}{4^3 \cdot 3!} + \frac{\pi^5}{4^5 \cdot 5!} - \frac{\pi^7}{4^7 \cdot 7!} + \cdots$$

(a) $$x^2 \sin(x) = x^2 \cdot \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^2 \cdot x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{(2n+1)!}$$

(b) $$\frac{\pi}{4} - \frac{\pi^3}{4^3 \cdot 3!} + \frac{\pi^5}{4^5 \cdot \cancel{3}5!} - \frac{\pi^7}{4^7 \cdot 7!} + \cdots$$

$$= \frac{(\pi/4)^1}{1!} - \frac{(\pi/4)^3}{3!} + \frac{(\pi/4)^5}{5!} - \frac{(\pi/4)^7}{7!} + \cdots$$

$$= \sin(\pi/4)$$

$$= 1/\sqrt{2}$$

2. Find the Maclaurin series for $f(x) = \dfrac{1}{(1-x)^2}$ by using the definition of a Maclaurin series.

$$f(x) = \frac{1}{(1-x)^2} \implies f(0) = \frac{1}{(1-0)^2} = 1$$

$$f'(x) = \frac{2}{(1-x)^3} \implies f'(0) = \frac{2}{(1-0)^3} = 2$$

$$f''(x) = \frac{3\cdot 2}{(1-x)^4} \implies f''(0) = \frac{3\cdot 2}{(1-0)^4} = 3\cdot 2$$

$$f'''(x) = \frac{4\cdot 3\cdot 2}{(1-x)^5} \implies f'''(0) = \frac{4\cdot 3\cdot 2}{(1-0)^5} = 4\cdot 3\cdot 2$$

$$\vdots$$

$$f^{(n)}(x) = \frac{(n+1)!}{(1-x)^{n+2}} \implies f^{(n)}(0) = (n+1)!$$

$$T = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{(n+1)!}{n!} x^n$$

$$= \sum_{n=0}^{\infty} (n+1) x^n$$