Name: Solutions. Section: 7 8

1. Find the Taylor polynomial T_2 of order 2 of $f(x) = x^{2/3}$ centered at $x = 8$.

$f(x) = x^{2/3} \Rightarrow f(8) = 8^{2/3} = 4$

$f'(x) = \frac{2}{3} x^{-1/3} \Rightarrow f'(8) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$f''(x) = -\frac{2}{9} x^{-4/3} \Rightarrow f''(8) = -\frac{2}{9} \cdot \frac{1}{2^4} = -\frac{1}{72}$

$f'''(x) = \frac{8}{27} x^{-7/3}$

$T_2(x) = 4 + \frac{1}{3}(x-8) + \frac{-1/72}{2!}(x-8)^2$

2. Use Taylor's inequality to find a bound on the error in using the approximation T_2 of f on the interval $|x - 8| \leq 2$.

$$\left| f'''(x) \right| = \left| \frac{8}{27} \cdot \frac{1}{x^{7/3}} \right| = \frac{8}{27} \cdot \frac{1}{|x|^{7/3}} \leq \frac{8}{27} \cdot \frac{1}{6^{7/3}}$$

$$\left| R_2(x) \right| \leq \frac{\frac{8}{27} \cdot \frac{1}{6^{7/3}} \left| x - 8 \right|^3}{3!}$$

$$\leq \frac{\frac{8}{27} \cdot \frac{1}{6^{7/3}} \cdot 2^3}{3!}$$

3. Solve the separable differential equation

$$\frac{dy}{dx} = y\cos(x).$$

$\frac{1}{y}\,dy = \cos x\,dx$

$\int \frac{1}{y}\,dy = \int \cos x\,dx$

$\ln|y| = \sin x + C$

$|y| = e^{\sin x + c} = e^{c}\cdot e^{\sin x}$

$y = \pm e^{c}\cdot e^{\sin x}$

since $y=0$ is a solution we can write

$y = A\cdot e^{\sin x}, \quad A \in \mathbb{R}.$

4. Solve the first-order linear equation

$$(x+1)y' + 3y = x, \quad x > -1.$$

$y' + \frac{3}{x+1}\,y = \frac{x}{x+1}$

$I = e^{\int \frac{3}{x+1}dx} = e^{3\cdot\ln|x+1|}$

$= |x+1|^3$

$= (x+1)^3$

Integrating factor

multiply eq'n by I.

$(x+1)^3 y' + 3(x+1)^2 y = x(x+1)^2$

$((x+1)^3 y)' = x(x^2+2x+1)$

$(x+1)^3 y = \int x^3 + 2x^2 + x\,dx$

$y = \frac{1}{(x+1)^3}\left[\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + C\right]$