Math 242 Exam 1 Practice Problems, Spring 2023

Name:

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
Total:	0	

- You have 75 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may not use notes or calculators on this exam.
- You do not need to simplify your answers.

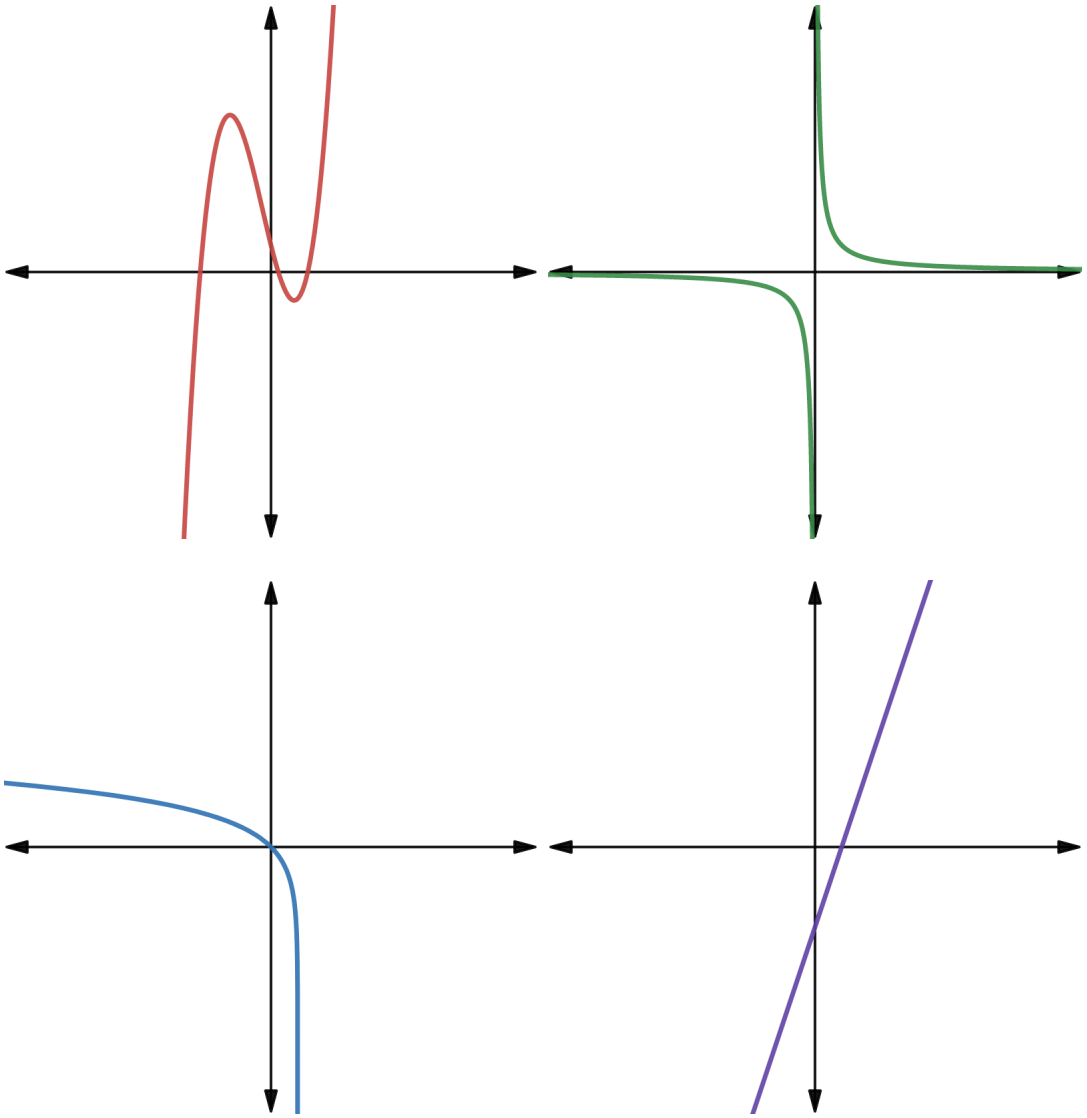
1. Evaluate the following

   (a) $\log_9(81)$, $\log_{\sqrt{2}} 8$, $\log_5(1/\sqrt{5})$, $\ln(e^2)$.

   (b) $\arcsin(1/2)$, $\tan^{-1}(-\sqrt{3})$, $\arccos(-1)$, $\sin(\arctan(10))$ (hint: draw a triangle)

2. Find the inverse of $f(x) = \ln\left(\frac{1-x}{1+x}\right)$

3. Determine if the following functions are one-to-one, for the ones that are sketch the graph of the inverse on the same plot.



4. If f is an invertible differentiable function with $f(2) = 14$ and $f'(14) = 3$, find df^{-1}/dx at $a = 14$.

5. The function $f(x) = e^x + x^2 - 1$ is invertible in a small interval around 0. Find $(f^{-1})'(0)$.

6. Differentiate the following functions.

   (a) $y = \ln\sqrt{x} + \log_{10}(x^2 + 1)$

(b) $y = e^{\cos x} + 2^{-x/3}$

(c) $y = \arctan(\arccos\sqrt{x}), \quad y = \sec^{-1}(3x)$

(d) $y = (4 + \sin x)^{\cot x}$.

7. Compute the following limits. Indicate where you use L'Hôpital's rule.

(a) $\lim_{x\to 1} \dfrac{x^{1/4} - 1}{\sqrt{x} - 1}, \quad \lim_{x\to \pi^-} \cot x, \quad \lim_{x\to\infty} \dfrac{|\sin x|}{x^2 + 1}$

(b) $\lim_{x\to 0^+} \ln \dfrac{1}{x}, \quad \lim_{x\to 2^+} \ln(x - 2), \quad \lim_{x\to\infty} \ln \dfrac{ex - 1}{x + 1},$

(c) $\lim_{x\to 0^+} \dfrac{4e^{1/x}}{e^{1/x} + 1}, \quad \lim_{x\to 0^+} e^x \ln x, \quad \lim_{x\to 0} e^{\arcsin x},$

(d) $\lim_{x\to\infty} \arctan(x), \quad \lim_{x\to \pi/2} \arccos(x),$

(e) $\lim_{x\to 0} \dfrac{\sin(3x)}{x}, \quad \lim_{x\to 0} \dfrac{\cos(2x) - 1}{\sin x}, \quad \lim_{x\to\infty} \dfrac{3^x - 1}{2^x - 1},$

(f) $\lim_{x\to 0^+} x \ln x, \quad \lim_{x\to 0} x^2 e^{-x},$

(g) $\lim_{x\to 0}(2\ln(2x) - \ln(x^2 + 1)),$

(h) $\lim_{x\to\infty} x^{1/\ln x}, \quad \lim_{x\to 0^+} (\tan x)^x, \quad \lim_{x\to 0^+} (1 + \dfrac{1}{x})^x \ ,$

8. Evaluate the following integrals.

(a) $\int x^{3/2} + \cos(x)\ dx, \quad \int \sec(x)\tan(x)\ dx, \quad \int \dfrac{1}{\sin^2(x)}\ dx$

(b) $\int \dfrac{\log_2(2x)}{x}\ dx, \quad \int \dfrac{\ln\ln x}{x \ln x}\ dx, \quad \int \dfrac{1}{x(\ln x)^3}\ dx, \quad \int \dfrac{\sec x}{(\ln(\sec x + \tan x))^{3/2}}\ dx,$

(c) $\int \dfrac{dx}{x^2 + 16}\ dx, \quad \int \dfrac{x}{x^2 + 16}\ dx, \quad \int \dfrac{1}{\sqrt{9 - x^2}}\ dx, \quad \int \dfrac{x}{\sqrt{9 - x^2}}\ dx,$

(d) $\int \dfrac{dx}{3x - 2} dx, \quad \int \dfrac{\sin x}{1 - \cos x}\ dx, \quad \int \dfrac{dx}{x \ln x} dx$

(e) $\int e^{-3x}\ dx, \quad \int \dfrac{e^{1/x}}{x^2}\ dx, \quad \int 7^{-t}\ dt,$

(f) $\int x \ln x\ dx, \quad \int x^3 \ln x\ dx, \quad \int \dfrac{\ln x}{x^2}\ dx,$

(g) $\int x \tan^{-1}(x)\ dx, \quad \int \arcsin(x)\ dx,$

(h) $\int xe^x\ dx, \quad \int xe^{3x}\ dx, \quad \int (x^2 - 5x + 3)e^x\ dx,$

(i) $\int x \sin(x)\ dx, \quad \int x \cos(2x)\ dx,$

(j) $\int e^{-x} \cos(x)\ dx,$

9. A population of baceria has an initial population of 1000, 2 hours later there is 2100. How long will it take for the population to reach 100,000.

10. A sample of an isotope initially has 100 micrograms. After 48 hours 20 micrograms remain. What is the half-life of the substance?