Math 242 Exam 3, Spring 2023

Name:

Section: 7 8

Question	Points	Score
1	0	
2	0	
3	0	
4	0	
Total:	0	

- You have 75 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work unless the problem states otherwise. You will get almost no credit for solutions that are not fully justified.
- You may not use notes or calculators on this exam.
- You do not need to simplify your answers.
- Good luck!

Homework	
Worksheets	
Quizzes	
Exam 1	
Exam 2	
Exam 3	
Total	

(i) Sequences $\{a_n\}$ are infinite lists of numbers. Make sure you practice your limit techniques.

(ii) Series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$ is the sum of a sequence. It is the limit of the sequence of n-th partial sums $s_n = a_1 + \cdots + a_n$.

(iii) A sequence is **not** the same as a series

$$\sum_{n=1}^{\infty} a_n \neq \lim_{n\to\infty} a_n$$

1. Find the limit of the following sequences, otherwise state that sequence diverges.

   (a) $\left\{\dfrac{2n^2}{3n^2-1}\right\}$

   (b) $\left\{\dfrac{\ln n + (-1)^n}{3n^2-1}\right\}$

   (c) $\left\{(-1)^{n+1}\dfrac{2n}{3n-1}\right\}$

   (d) $\left\{\left(\dfrac{4}{5}\right)^n\right\}$

   (e) $\{\sqrt[n]{n}\}$

2. Which of the following series converge, if the series converges find its sum.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{5^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{9^{n-1}}{2^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{7^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n}{3^n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (\arctan(n+1) - \arctan(n+2))$

3. Determine which of the following series converge or diverge. Clearly state the tests you are using and give full reasoning.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{3n^2+2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+n}{4^n-n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

(e) $\displaystyle\sum_{n=3}^{\infty} \frac{2}{n\ln n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[6]{n}}$

(g) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n^3)}{n^2+\sqrt{n}}$

(i) $\displaystyle\sum_{n=1}^{\infty} (-1)^n|\sin(n)|$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

(k) $\displaystyle\sum_{n=1}^{\infty} (3\cos(1/n))^n$

(l) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

(m) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+n}{n}\right)^{2n}$

(n) $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{2}{n}\right)^{2n^2}$

(o) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^5}$

(p) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(q) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

(r) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

(s) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}-1}$

(t) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3-2n^2-5}$

(u) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

(v) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$

(w) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n)$

(x) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)}{n\sqrt{n}}$

4. Determine if the following series converge absolutely, converge conditionally, or if they diverge.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln n}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{n\ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n^3)}{n^2+\sqrt{n}}$

(d) $\sum_{n=1}^{\infty}(-1)^n \frac{n^n}{n!}$

(e) $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{\ln}{n^3}$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$