

Math 244 Exam 2, Spring 2022

Name:

Question	Points	Score
1	7	
2	13	
3	17	
4	13	
5	0	
Total:	50	

- You have 50 minutes to complete this exam.
- All work must be your own.
- You may use a 3x5 notecard with notes that you may turn in for extra credit.
- You must show all your work. You will get almost no credit for solutions that are not fully justified.
- Answer the questions in the space provided. Use the back of the page for scratch work or if you require additional space for your answers. Clearly indicate what is a solution, and what is scratch work.
- No electronic devices are authorized with the exception of a scientific calculator.
- Good luck!

1. (7 points) Evaluate $\iiint_B \frac{xz^3}{y^2} dV$ where $B = [0, 2] \times [1, 2] \times [1, 3]$.

2. (13 points) Find the mass of the solid hemisphere of radius 1 above the xy -plane with density $\rho(x, y, z) = z^2(x^2 + y^2 + z^2)^{1/2}$

3. (17 points) The rotational kinetic energy of a solid object rotating about an axis is given by

$$KE_{rot} = \frac{1}{2}I\omega^2,$$

where I is the moment of inertia about the axis of rotation and ω is the angular velocity.

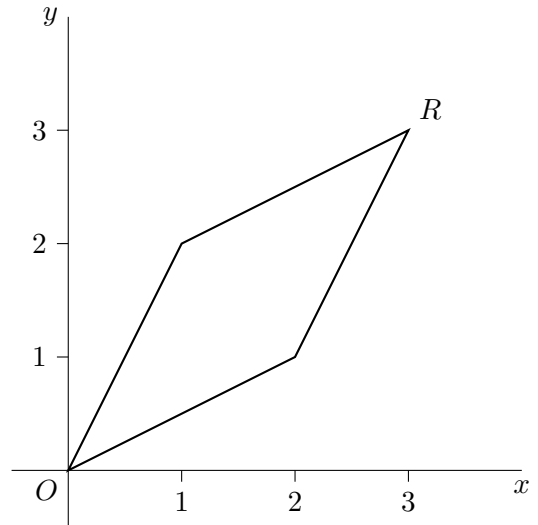
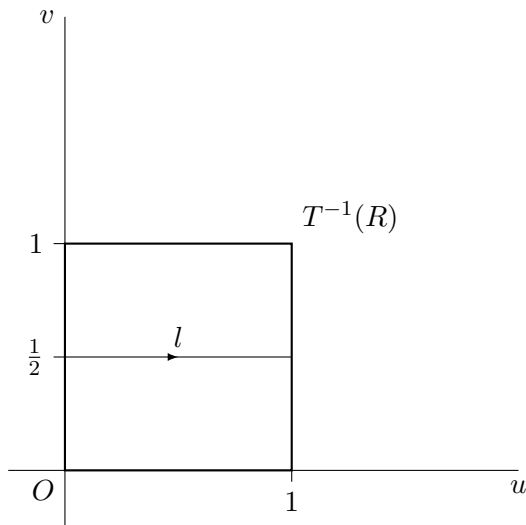
Suppose a homogeneous right circular cylinder with height h and radius R is rotating once per second about its axis. Find the rotational kinetic energy of this system in terms of h , R and the density. *Hint:* put the axis of the cylinder on one of the coordinate axes.

4. Consider the transformation

$$T(u, v) = (u + 2v, 2u + v).$$

- (a) (3 points) Describe the image $T(l)$ of the line segment l (see below) under the transformation, and sketch it in the region R below.
- (b) (3 points) Find the Jacobian of the transformation.
- (c) (7 points) Below you are given a region R and its preimage $T^{-1}(R)$. Use a change of variables to compute

$$\iint_R 2y - x \, dA.$$



(continue problem 3 here)

5. (5 points (bonus)) Find the volume of the ellipsoid

$$\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1.$$