# MATH 244 Summer 2019
# Exam 2

Name: ______________________________

Instructions:

- You have 80 minutes to complete this exam.
- You may use a scientific calculator and a 3x5 notecard/cheat sheet during the exam.
- You must show all of your work. Answers which are cryptic or have no supporting evidence will most likely not receive full credit. When in doubt, ask.
- Please be organized! Answer questions in the space provided as neatly as possible. If you run out of room, continue on a piece of scratch paper and make a clear note of it.

| Question | Points | Score |
|---|---|---|
| 1 | 8 | |
| 2 | 10 | |
| 3 | 9 | |
| 4 | 13 | |
| 5 | 6 | |
| 6 | 10 | |
| 7 | 12 | |
| 8 | 9 | |
| Total: | 77 | |

1. (8 points) Compute the center of mass of a spring with constant density $\delta$ given by the parametrization

$$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k},$$

where $0 \leq t \leq 4\pi$.

2. (10 points) Parametrize the following curves.

   (a) The straight line segment from $(0, 19, 3)$ to $(1000, 2, 4)$.

   (b) The portion of the circle $x^2 + (y - 8)^2 = 4$ moving clockwise from the point $(0, 10)$ to $(0, 6)$.

3. (9 points) Compute the flux across the unit circle of the vector field $\mathbf{F} = y\mathbf{i} + 2y^2\mathbf{j}$.

4. (13 points) Evaluate the integral

$$\iint_R \frac{x-2y}{3x-y}\,dA,$$

where $R$ is the parallelogram bounded by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, and $3x-y=8$ using an appropriate transformation.

5. (6 points) Determine if the following vector fields are conservative

   (a) $yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$.

   (b) $-y\mathbf{i} + x\mathbf{j}$.

6. (10 points) Find the potential function of the vector field $y \sin z\mathbf{i} + x \sin z\mathbf{j} + xy \cos z\mathbf{k}$.

7. (12 points) Compute the flow of the vector field $\mathbf{F} = (x+y)\mathbf{i}+(x^2+y^2)\mathbf{j}$ along the line segment from $(0,0)$ to $(0,1)$ followed by the line segment from $(0,1)$ to $(1,1)$.

8. (9 points) Let $\nabla(e^z)$ be a vector field representing force. Compute the work over the following curves.

   (a) The line segment connecting the point $(3, 1, \ln 1)$ to the point $(-1, 5, \ln 5)$.

   (b) Counterclockwise around the circle cut from the cone $z = \sqrt{x^2 + y^2}$ by the plane $z = 7$.

   (c) Around *any* curve in the $xy$-plane.