MATH 244 Summer 2019
Final Exam

Name: ______________________________

Instructions:

- You have 80 minutes to complete this exam.
- You may use a scientific calculator and a 8.5x11 cheat sheet during the exam (one side)
- You must show all of your work. Answers which are cryptic or have no supporting evidence will most likely not receive full credit. When in doubt, ask.
- Please be organized! Answer questions in the space provided as neatly as possible. If you run out of room, continue on a piece of scratch paper and make a clear note of it.

Question	Points	Score
1	10	
2	13	
3	7	
4	26	
5	6	
6	18	
7	14	
Total:	94	

1. (10 points) Compute the counterclockwise circulation of $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$ around the boundary of the top half of the unit circle in the xy-plane: $x^2 + y^2 \leq 1$ and $y \geq 0$.

2. (13 points) Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ cut by the planes $z = 1$ and $z = 4$.

3. Consider the integral

$$\int_0^2 \int_0^{2-2y} \int_0^{2-2y-z} f(x,y,z)\,dx\,dz\,dy.$$

(a) (4 points) Sketch the region of integration.

(b) (3 points) Change the order of integration to $dzdydx$.

4. Let S be the upper half of the unit sphere oriented outward S: $x^2 + y^2 + z^2 = 1$ and $z \geq 0$. Let $\mathbf{F} = -y\mathbf{i} + z\mathbf{k}$.

   (a) (18 points) Compute the outward flux of $\nabla \times \mathbf{F}$ across the surface S.

   (b) (8 points) Use Stokes' theorem to verify your answer from the previous part.

5. Find the work done be $\mathbf{F} = \nabla(2^{x+y+z})$ in moving an object along the following curves.

(a) (3 points) Along the line segment connecting $(1, 1, 1)$ to the point $(2, 2, 2)$.

(b) (3 points) Once around the ellipse $x^2 + y^2/4 = 1$.

6. Let E be the portion of the solid ball of radius 2 in the first octant E: $x^2 + y^2 + z^2 \leq 4$.
   (a) (10 points) Find the volume of E.
   (b) (8 points) Compute the outward flux of $\mathbf{F} = x^2\mathbf{i} - 2xy\mathbf{j} + 3z\mathbf{k}$ across the boundary of E.

7. (14 points) Evaluate the integral $\iint_R (x+y)^2 e^{x-y} dxdy$ by using the transformation $u = x + y$ and $v = x - y$ over the region R bounded by the lines $x + y = 1$, $x + y = 4$, $x - y = -1$, and $x - y = 1$.