Math 244 Summer 2020 Midterm

July 10, 2020

- Please read each problem carefuly and follow directions.
- You may answer all the question on ths exam, or on a separate medium.
- You have 24 hours to complete this exam. It is due Sunday 7/12/2020 at 12am.
- Authorized aid: textbook, notes, and you may use a scientific calculator. All other aid is unauthorized: graphing calculators, google, friends and family, SymboLab, MatLab, etc.
- Show all of your work. Leaps in logic without justification will not receive credit.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

Name and section: ______________________________

1. (10 points) Let

$$I = \int_0^1 \int_0^{-x} \int_0^{2-2x-2y} (x+y)\,dz\,dy\,dx.$$

(a) Sketch the region of integration.

(b) Change the order of integration of I to $dydxdz$.

2. (10 points) Use the techniques of this chapter to verify that the volume of a sphere of radius R is $4\pi R^3$.

3. (10 points) Let $E \subset \mathbb{R}^3$ be the solid bounded above by the cone $z = 1 - r$ and below by the plane $z = 0$. Find the centroid of E. Hint: this question can be answered by solving only one integral.

4. (10 points) The relationship between spherical coordinates and cylidrical coordinates is given by

$$\begin{aligned} r &= \rho \sin \phi \\ \theta &= \theta \\ z &= \rho \cos \phi. \end{aligned}$$

Viewed as a transformation $T(\rho, \phi, \theta) = (r, \theta, z)$, compute its Jacobian $\frac{\partial(r,\theta,z)}{\partial(\rho,\phi,\theta)}$.

5. (10 points) Perform a change of variables on the integral

$$\iint_R \sqrt{x^2 - xy - 2y^2}\,dA,$$

where R is the triangle with vertices $(0,0)$, $(1,0)$, and $(0,-1)$. Write your answer as an iterated integral in the new coordinates, you do not need to evaluate the integral.

6. (10 points) Let X and Y be continuous random variables with probability density function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$f(x, y) = \begin{cases} Ce^{-x-y} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find C.

(b) What is $P((X, Y) \in [0, 1] \times [0, 3])$?

7. (10 points) Let $E \subset \mathbb{R}^3$ be the unit ball with its center at the point $(0, 0, 1)$, that is, all points satisfying the inequality $x^2 + y^2 + (z - 1)^2 \leq 1$. Rewrite the integral $\iiint_E f(x, y, z)dxdydz$ as an iterated integral in spherical coordinates.