

#22.

$$\int_0^1 \int_0^2 y e^{x-y} dx dy$$

$$= \int_0^1 \int_{-y}^{2-y} y e^u du dy$$

$$= \int_0^1 y e^u \Big|_{-y}^{2-y} dy$$

$$= \int_0^1 y e^{2-y} - y e^{-y} dy$$

$$= \left[-y e^{2-y} - e^{2-y} \right]_0^1$$

$$- \left[y e^{-y} - e^{-y} \right]_0^1$$

$$= \left[-e - e - (0 - e^2) \right] - \left[-e^{-1} - e^{-1} - (0 - 1) \right]$$

$$= -2e + e^2 + 2e^{-1} - 1 \approx 1.7$$

$$u = x - y$$

$$du = dx$$

$$u(0) = 0 - y$$

$$u(2) = 2 - y$$

$$\begin{array}{l} y \quad / \quad + \quad e^{2-y} \\ 1 \quad / \quad - \quad -e^{2-y} \\ 0 \quad / \quad e^{2-y} \\ \\ y \quad / \quad + \quad e^{-y} \\ 1 \quad / \quad - \quad -e^{-y} \\ 0 \quad / \quad e^{-y} \end{array}$$

32.

$$\int_0^1 \int_0^1 \frac{x}{1+yx} dy dx$$

$$u(y) = 1+yx$$

$$du = x dy$$

$$u(0) = 1$$

$$u(1) = 1+x$$

$$= \int_0^1 \int_1^{1+x} \frac{du}{u} dx$$

$$= \int_0^1 \ln|u| \Big|_1^{1+x} dx$$

$$= \int_0^1 \ln|1+x| - \ln(1) dx$$

$$= \int_0^1 \ln(1+x) dx$$

$$u = \ln(1+x) \quad dw = dx$$

$$du = \frac{1}{1+x} dx \quad v = x$$

$$= x \cdot \ln(1+x) \Big|_0^1 - \int_0^1 \frac{x}{1+x} dx$$

$$\frac{x}{1+x} = \frac{x+1-1}{1+x} = 1 - \frac{1}{1+x}$$

$$= \ln(2) - \int_0^1 1 - \frac{1}{1+x} dx$$

$$= \ln(2) - \left[x - \ln(1+x) \right]_0^1$$

$$= \ln(2) - [1 - \ln(2) - 0]$$

$$= -1 + 2\ln(2)$$

$$= \ln 4 - 1 \approx 0.4$$

36.

