#6. $\int_{\pi/2}^{\pi} \int_{0}^{2\sin\theta} r \, dr \, d\theta$

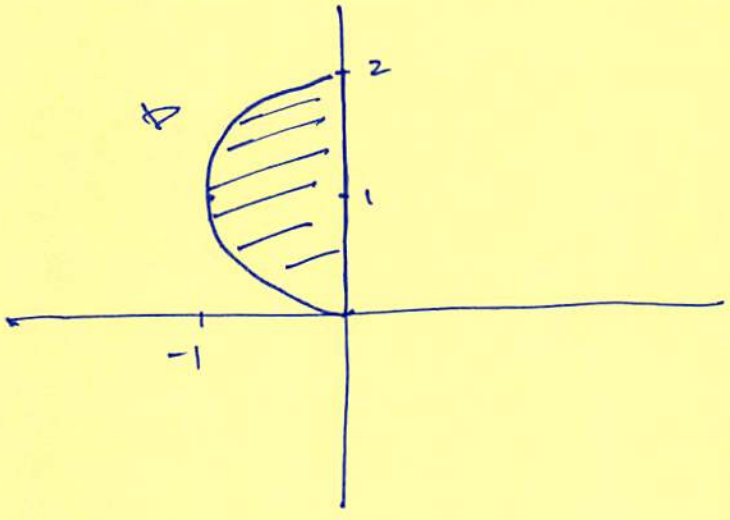
- $\frac{\pi}{2} \le \theta \le \pi \longrightarrow$ Region in Quadrant II

- $r = 2\sin\theta \implies r^2 = 2r\sin\theta$

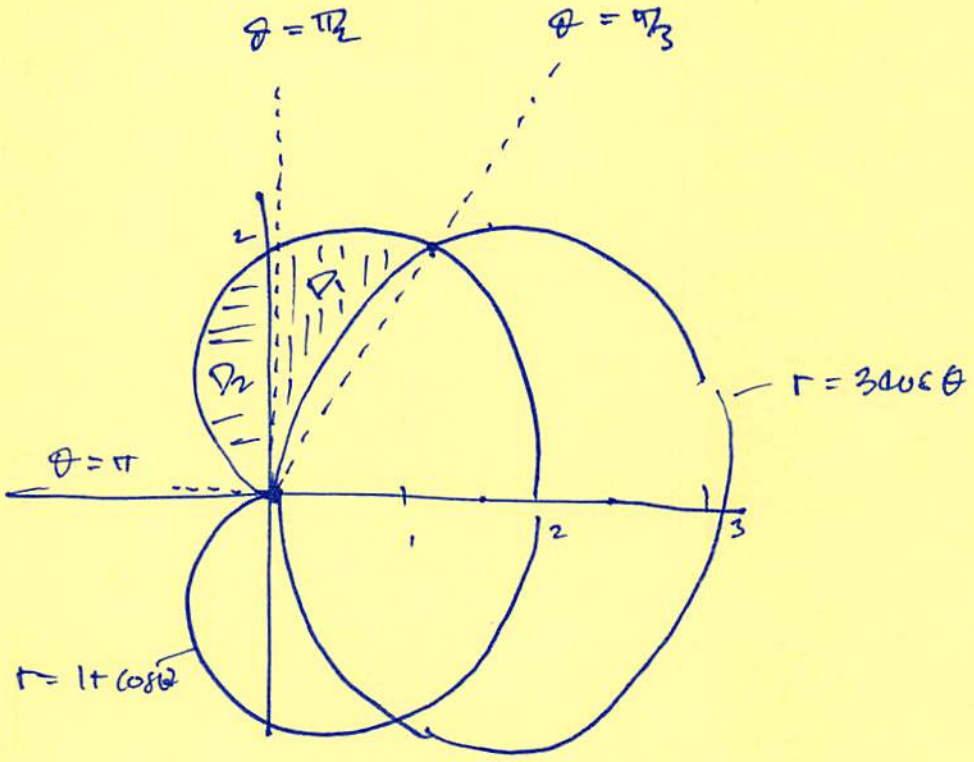
$\implies x^2 + y^2 = 2y$

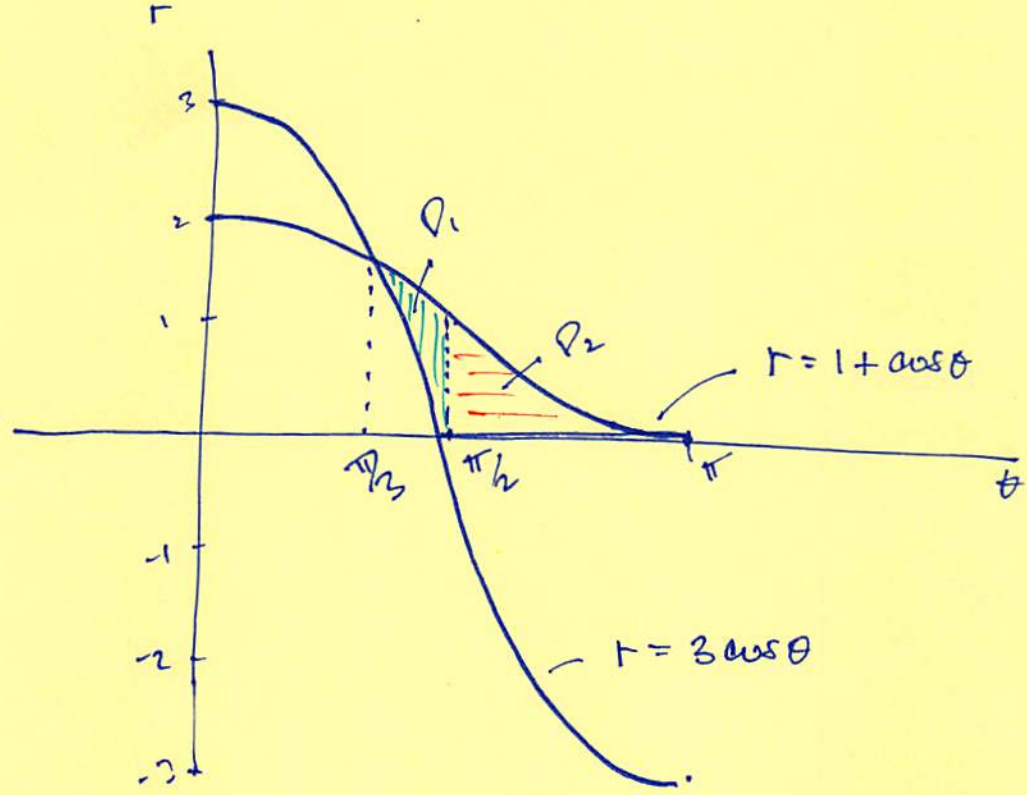
$\implies x^2 + (y-1)^2 = 1$

circle of radius 1 shifted up 1 unit.



18.


θ = π/2
θ = π/3
r = 3 cos θ
θ = π
r = 1 + cos θ
Ω1
Ω2
1
2
3


r
θ
Ω1
Ω2
r = 1 + cos θ
r = 3 cos θ
π/3
π/2
π
3
2
1
-1
-2
-3

$$A(D) = \iint_D 1\,dA = \iint_{D_1} 1\cdot dA + \iint_{D_2} 1\,dA$$

One at a time.

$$\iint_{D_1} 1\,dA = \int_{\pi/3}^{\pi/2}\int_{3\cos\theta}^{1+\cos\theta} r\,dr\,d\theta$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi/2} r^2 \Big|_{3\cos\theta}^{1+\cos\theta} d\theta$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi/2} 1 + 2\cos\theta + \cos^2\theta - 9\cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi/2} 1 + 2\cos\theta - 8\left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\int_{\pi/3}^{\pi/2} -3 + 2\cos\theta - 4\cos 2\theta\,d\theta$$

$$= \frac{1}{2}\left[-3\theta + 2\sin\theta - 2\sin 2\theta \right]_{\pi/3}^{\pi/2}$$

$$= \frac{1}{2}\left[-\frac{3\pi}{2} + 2 - 0 - \left(-\pi + \sqrt{3} - \sqrt{3} \right) \right]$$

$$= \frac{1}{2}\left[2 - \frac{\pi}{2} \right]. \quad = 1 - \frac{\pi}{4}$$

Next,

$$\iint_{D_2} 1 \, dA = \int_{\pi/2}^{\pi} \int_{0}^{1+\cos\theta} r \, dr \, d\theta$$

$$= \frac{1}{2} \int_{\pi/2}^{\pi} r^2 \Big|_0^{1+\cos\theta} d\theta .$$

$$= \frac{1}{2} \int_{\pi/2}^{\pi} 1 + 2\cos\theta + \cos^2\theta \, d\theta$$

$$= \frac{1}{2} \int_{\pi/2}^{\pi} 1 + 2\cos\theta + \frac{1}{2}\left(\cancel{1} + \cos 2\theta \right) d\theta$$

$$= \frac{1}{2}\int_{\pi/2}^{\pi} \frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta \, d\theta$$

$$= \frac{1}{4}\left[3\theta + 4\sin\theta + \frac{1}{2}\sin 2\theta\right]_{\pi/2}^{\pi}$$

$$= \frac{1}{4}\left[3\pi + 0 - \left(\frac{3\pi}{2} + 4\right)\right]$$

$$= \frac{1}{4}\frac{3\pi}{2} - 1 \quad = \quad \frac{3\pi}{8} - 1$$

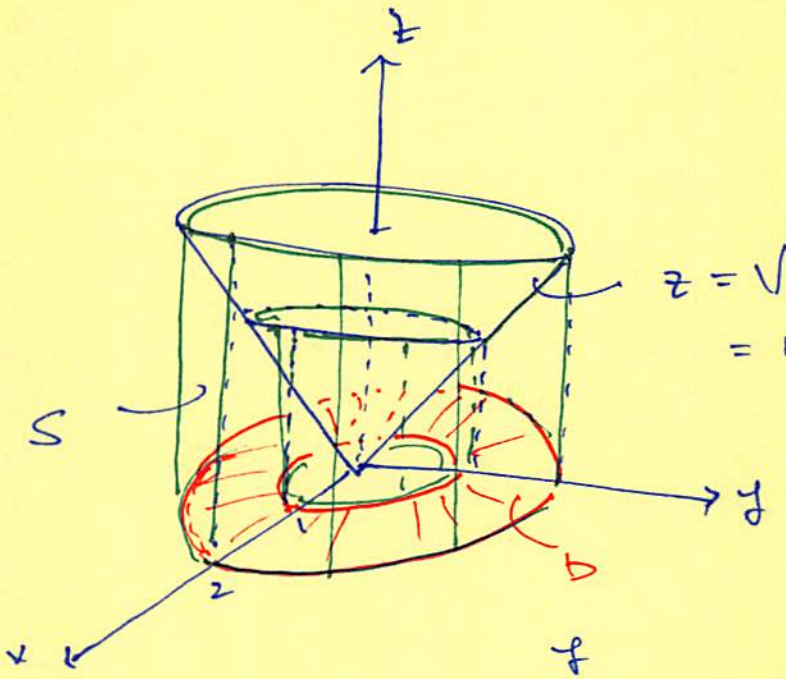
$\therefore$ total Area $= 2 \cdot A(D)$

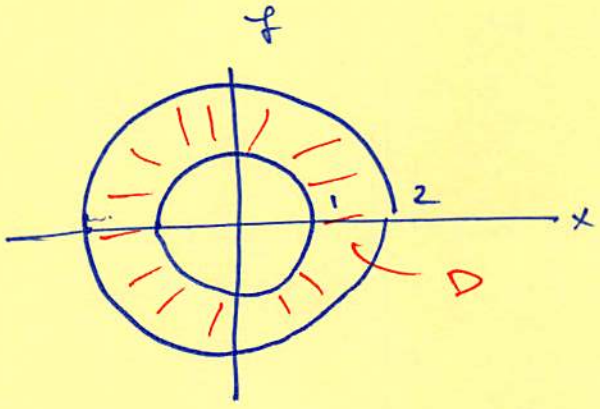
$$= 2 \cdot \left[\frac{1}{2}\left(2 - \frac{\pi}{2}\right) + \frac{1}{4}\cdot\frac{3\pi}{2} - 1\right]$$

$$= 2 - \frac{\pi}{2} + \frac{3\pi}{4} - 2$$

$$= \frac{\pi}{4}$$

#20.





$D: \quad 0 \le \theta \le 2\pi$

$1 \le r \le 2$

$$V(S) = \iint_D \sqrt{x^2+y^2}\, dA = \int_0^{2\pi} \int_1^2 r \cdot r\, dr\, d\theta$$

$$= 2\pi \cdot \frac{r^3}{3}\Big|_1^2$$

$$= \frac{14\pi}{3}$$