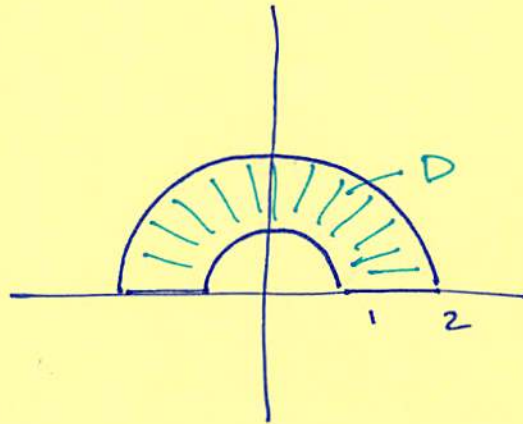


15.4  
# 14



$$\rho(x,y) = \frac{k}{\sqrt{x^2+y^2}}, \quad k \in \mathbb{R}$$

by symmetry  $\bar{x} = 0$ .

$$\begin{aligned} m &= \iint_D \rho(x,y) \, dA = \int_0^\pi \int_1^2 \frac{k}{r} \cdot r \, dr \, d\theta \\ &= k \cdot \int_0^\pi d\theta \cdot \int_1^2 dr \\ &= \pi k. \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{M_x}{m} = \frac{1}{\pi k} \cdot \iint_D y \cdot \rho(x,y) \, dA \\ &= \frac{1}{\pi k} \cdot \int_0^\pi \int_1^2 r \sin \theta \cdot \frac{k}{r} \cdot r \, dr \, d\theta \end{aligned}$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin \theta \, d\theta \cdot \int_1^2 r \, dr$$

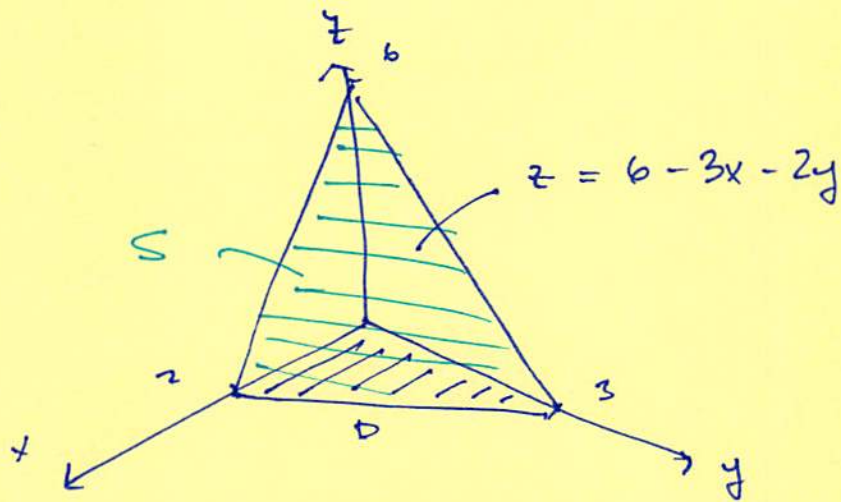
$$= \frac{1}{\pi} \cdot \left[ -\sin \theta \right]_0^{\pi} \cdot \left[ \frac{r^2}{2} \right]_1^2$$

$$= \frac{1}{\pi} \cdot 2 \cdot \frac{3}{2}$$

$$= \frac{3}{\pi}$$

15.5

#3.



$$\frac{\partial z}{\partial x} = -3 \quad \Rightarrow \quad \frac{\partial z^2}{\partial x^2} = 9$$

$$\frac{\partial z}{\partial y} = -2 \quad \Rightarrow \quad \frac{\partial z^2}{\partial y^2} = 4$$

$$\Rightarrow \sqrt{\frac{\partial z^2}{\partial x^2} + \frac{\partial z^2}{\partial y^2} + 1} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

$$\begin{aligned} A(S) &= \iint_D \sqrt{\frac{\partial z^2}{\partial x^2} + \frac{\partial z^2}{\partial y^2} + 1} \, dA = \iint_D \sqrt{14} \, dA \\ &= \sqrt{14} \cdot \iint_D 1 \cdot dA \\ &= \sqrt{14} \cdot A(D) \\ &= \sqrt{14} \cdot \frac{1}{2} \cdot 2 \cdot 3 \\ &= 3\sqrt{14} \end{aligned}$$