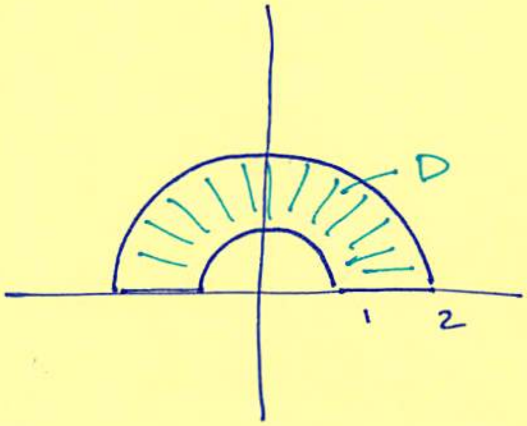
15.4

14



$$\rho(x,y) = \frac{k}{\sqrt{x^2+y^2}}, \quad k \in \mathbb{R}$$

by symmetry $\bar{x} = 0$.

$$m = \iint_D \rho(x,y)\, dA = \int_0^{\pi}\int_1^2 \frac{k}{r} \cdot r\, dr\, d\theta.$$

$$= k \cdot \int_0^{\pi} d\theta \cdot \int_1^2 dr$$

$$= \pi k.$$

$$\bar{y} = \frac{M_x}{m} = \frac{1}{m} \cdot \iint_D y \cdot \rho(x,y)\, dA$$

$$= \frac{1}{\pi k} \cdot \int_0^{\pi}\int_1^2 r\sin\theta \cdot \frac{k}{r} \cdot r\, dr\, d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin\theta \, d\theta \cdot \int_1^{2} r \, dr$$

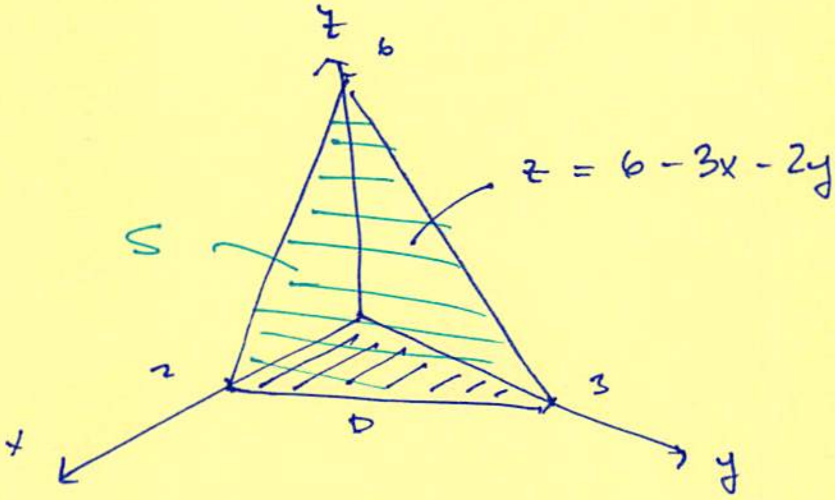
$$= \frac{1}{\pi} \cdot \Big[-\sin\theta \Big]_0^{\pi} \cdot \Big[\frac{r^2}{2} \Big]_1^{2}$$

$$= \frac{1}{\pi} \cdot 2 \cdot \frac{3}{2}$$

$$= \frac{3}{\pi}$$

15.5

#3.



$$\frac{\partial z}{\partial x} = -3 \implies \frac{\partial z}{\partial x}^2 = 9$$

$$\frac{\partial z}{\partial y} = -2 \implies \frac{\partial z}{\partial y}^2 = 4$$

$$\implies \sqrt{\frac{\partial z}{\partial x}^2 + \frac{\partial z}{\partial y}^2 + 1} = \sqrt{9+4+1} = \sqrt{14}$$

$$A(S) = \iint_D \sqrt{\frac{\partial z}{\partial x}^2 + \frac{\partial z}{\partial y}^2 + 1}\; dA = \iint_D \sqrt{14}\; dA$$

$$= \sqrt{14} \cdot \iint_D 1 \cdot dA$$

$$= \sqrt{14} \cdot A(D)$$

$$= \sqrt{14} \cdot \frac{1}{2} \cdot 2 \cdot 3$$

$$= 3\sqrt{14}$$