$y = x+4$

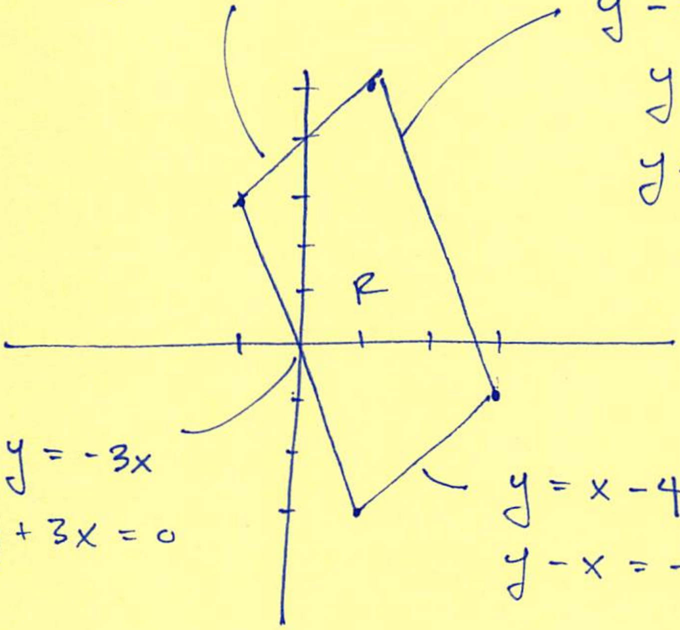
$y - x = 4$

#14.

$y - 5 = -3(x-1)$

$y = -3x + 8$ 8

$y + 3x = 8$



$y = -3x$

$y + 3x = 0$

$y = x - 4$

$y - x = -4$

$$T: \quad x = \tfrac{1}{4}(u+v) \qquad \text{find } T^{-1}$$
$$\quad\; y = \tfrac{1}{4}(v-3u)$$

$$\Longrightarrow \quad x - y = \tfrac{1}{4}\big(u+v-(v-3u)\big) = u$$

back sub

$$x = \tfrac{1}{4}\big((x-y)+v\big)$$

$$\Longrightarrow \quad 4x = x - y + v$$

$$\Longrightarrow \quad v = y + 3x$$

$$\therefore \; T^{-1}: \quad u = x - y$$
$$\qquad\qquad v = y + 3x$$

Find $T^{-1}(R)$:

$y - x = -4 \implies u = 4$

$y - x = 4 \implies u = -4$

$y + 3x = 8 \implies v = 8$

$y + 3x = 0 \implies v = 0$

Find Jacobian:

$$J(T) = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 1/4 & 1/4 \\ -3/4 & 1/4 \end{vmatrix}$$

$$= 1/16 + 3/16$$

$$= 1/4$$

$$\iint_R (4x+8y)\,dA = \int_0^8 \int_{-4}^4 \left(4\cdot\left[\frac{1}{4}(u+v)\right] + 8\cdot\left[\frac{1}{4}(v-3u)\right]\right) \cdot \left|\,1/4\,\right| \,du\,dv$$

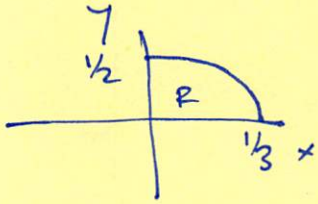
$$= \frac{1}{4}\int_0^8 \int_{-4}^4 u+v+2v-6u \;du\,dv$$

$$= \frac{1}{4}\int_0^8 \int_{-4}^4 3v-5u \;du\,dv$$

$$= \frac{1}{4}\left[3\cdot\frac{v^2}{2}\Big|_0^8 \cdot 8 - 5\cdot\frac{u^2}{2}\Big|_{-4}^4 \cdot 8\right]$$

$$= 3\cdot 8^2 - 5\left(4^2-(-4)^2\right)$$

$$= 192$$

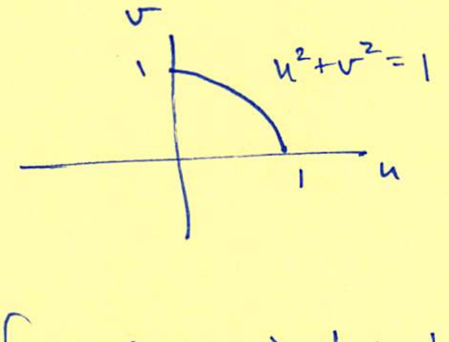
#26. $\iint_R \sin(9x^2+4y^2)\,dA$, R: 

$$= \iint_R \sin\left((3x)^2+(2y)^2\right)\,dA$$

Let $u = 3x$ and $v = 2y$

$$\longrightarrow \underbrace{x = \tfrac{1}{3}u \quad \text{and} \quad y = \tfrac{1}{2}v}_{T}$$

$$J(T) = \begin{vmatrix} 1/3 & 0 \\ 0 & 1/2 \end{vmatrix} = 1/6$$

$T^{-1}(R)$: 

$$= \iint_{T^{-1}(R)} \sin(u^2+v^2)\cdot|1/6|\cdot du\,dv$$

$$= \int_0^{\pi/2}\int_0^1 \sin(r^2)\cdot \tfrac{1}{6}\, r\, dr\, d\theta$$

$$= \pi/2 \cdot 1/6 \cdot \int_0^1 \sin(r^2) \cdot r \, dr$$

$u = r^2$ $\quad u(0) = 0$

$du = 2r\,dr$ $\quad u(1) = 1$

$$= \pi/24 \int_0^1 \sin(u) \, du$$

$$= \frac{\pi}{24}\left(\sin(1) - 0\right)$$

$$= \frac{\sin(1)\,\pi}{24}$$