16.1

#29. $\nabla f = 2x\vec{i} + 2y\vec{j}$, II

#30. $\nabla f = (2x+y)\vec{i} + x\vec{j}$, IV

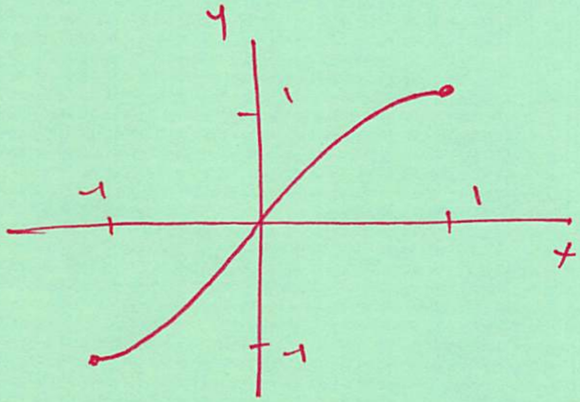
#31. $\nabla f = 2(x+y)\vec{i} + 2(x+y)\vec{j}$, II

#32. $\nabla f = \cos\sqrt{x^2+y^2} \cdot \left(\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x\right)\vec{i}$

$+ \cos\sqrt{x^2+y^2} \cdot \left(\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y\right)\vec{j}$. I

16.2

#6.



$C: \vec{r}(t) = t^3\vec{i} + t\vec{j}$

$-1 \le t \le 1$

$\vec{r}\,'(t) = 3t^2\vec{i} + \vec{j}$

$\longrightarrow x'(t) = 3t^2$

$f(x,y) = e^x \implies f(\vec{r}(t)) = f(t^3, t) = e^{t^3}$

$$\int_C e^x\,dx = \int_{-1}^{1} e^{t^3} \cdot 3t^2\,dt$$

$u = t^3 \qquad u(-1) = -1$

$du = 3t^2\,dt \qquad u(1) = 1$

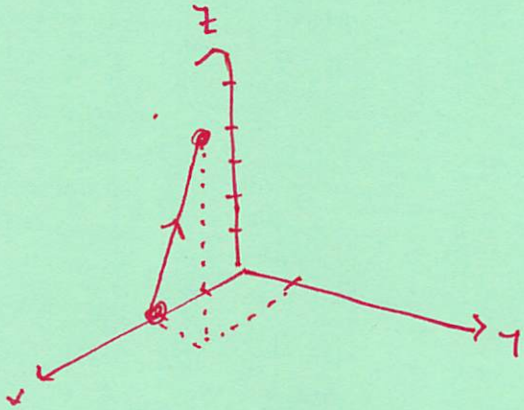
$$= \int_{-1}^{1} e^u\,du$$

$$= e^u \Big|_{-1}^{1}$$

$$= e^1 - e^{-1}.$$

#42. $\vec{x} = \langle x, y, z \rangle$

$$\vec{F} = \frac{K\vec{x}}{\|\vec{x}\|^3}$$



$$\vec{r}(t) = (1-t)\langle 2,0,0\rangle + t\langle 2,1,5\rangle$$

$$= \langle 2-2t, 0, 0\rangle + \langle 2t, t, 5t\rangle$$

$$= \langle 2, t, 5t\rangle, \quad 0 \le t \le 1$$

$$\vec{F}(\vec{r}(t)) = K \cdot \frac{\langle 2, t, 5t\rangle}{\sqrt{4 + t^2 + 25t^2}} = \frac{K \cdot \langle 2, t, 5t\rangle}{\sqrt{4 + 26t^2}}$$

$$\vec{r}\,'(t) = \langle 0, 1, 5\rangle$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \frac{K}{\sqrt{4+26t^2}}\left(0 + t + 25t\right)$$

$$= \frac{26tK}{\sqrt{4+26t^2}}$$

$$\text{work} = \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_0^1 \frac{26 K t}{\sqrt{4 + 26 t^2}}\, dt$$

$u = 4 + 26 t^2$

$du = 2 \cdot 26 t \, dt$

$u(0) = 4$

$u(1) = 30$

$$= \int_4^{30} \frac{2K\, du}{\sqrt{u}}$$

$$= 4 u^{1/2} \Big|_4^{30}$$

$$= 4 \left(\sqrt{30} - 2 \right)$$